Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 13 November 2017 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8817 – 7301

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

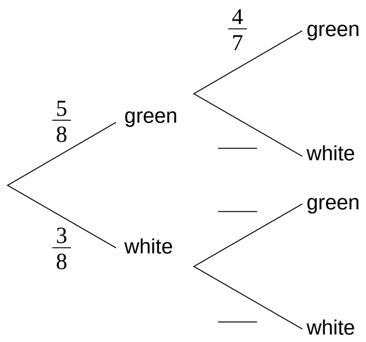
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

(a) Complete the following tree diagram. [3]



(b) Find the probability that exactly one of the selected balls is green. [3]

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2. [Maximum mark: 6]

In an arithmetic sequence, the first term is 8 and the second term is 5.

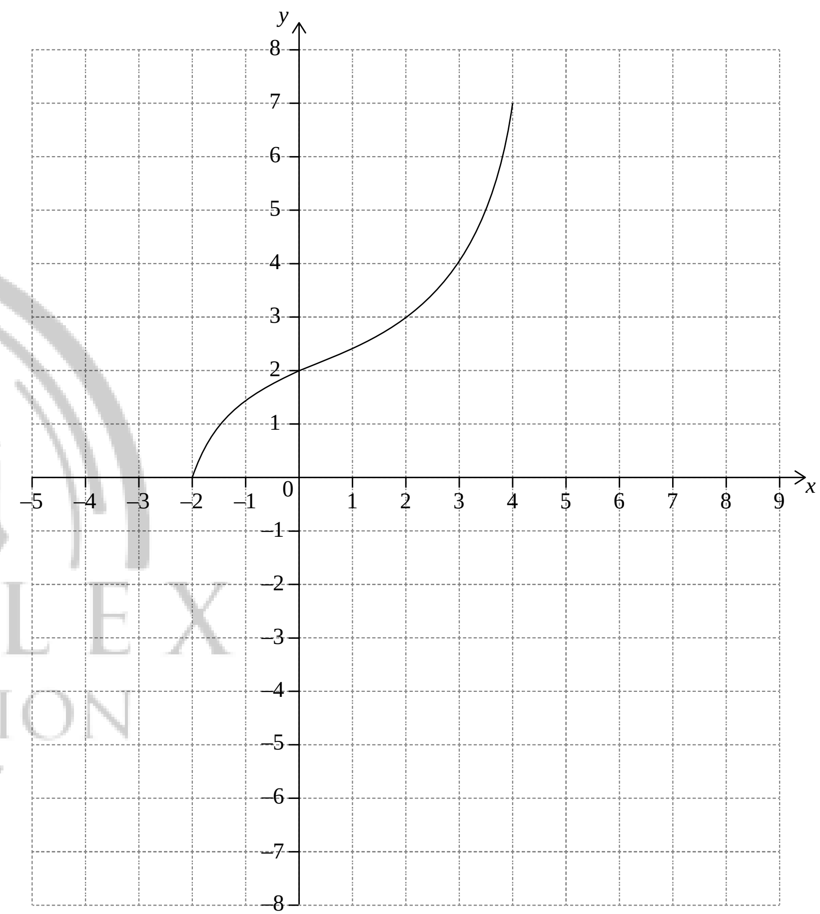
(a) Find the common difference. [2]

(b) Find the tenth term. [2]

(c) Find the sum of the first ten terms. [2]

3. [Maximum mark: 6]

The following diagram shows the graph of a function f, with domain $-2 \le x \le 4$.



The points $(-2, 0)$ and $(4, 7)$ lie on the graph of f.

(This question continues on the following page)

Turn over

(Question 3 continued)

(a) Write down the range of f. [1]

(b) Write down

(i) $f(2)$;

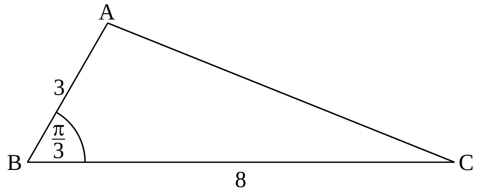
(ii) $f^{-1}(2)$. [2]

(c) On the grid opposite, sketch the graph of f^{-1}. [3]

4. [Maximum mark: 7]

The following diagram shows triangle ABC, with $AB = 3\text{ cm}$, $BC = 8\text{ cm}$, and $A\hat{B}C = \frac{\pi}{3}$.

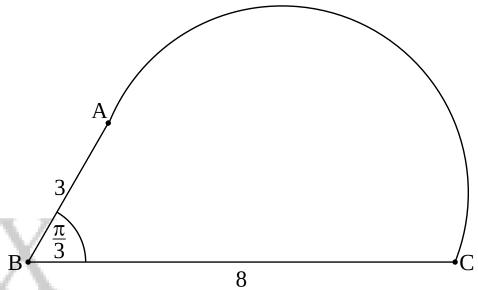
diagram not to scale



(a) Show that $AC = 7\text{ cm}$. [4]

(b) The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle.

diagram not to scale



Find the exact perimeter of this shape. [3]

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(Question 4 continued)

5. [Maximum mark: 6]

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

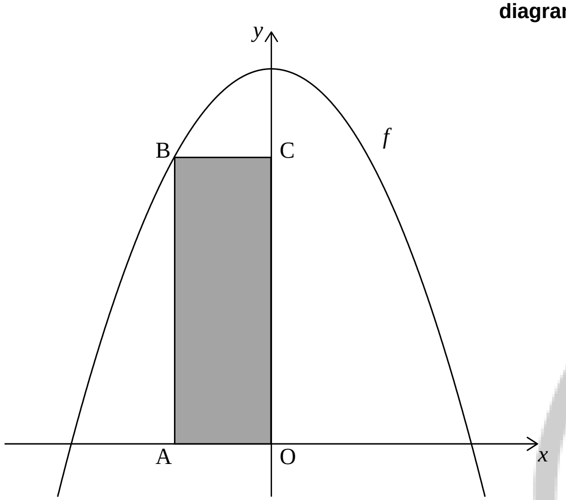
(a) Find $(g \circ f)(x)$. [2]

(b) Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4]

6. [Maximum mark: 7]

Let $f(x) = 15 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis.

diagram not to scale



Find the x-coordinate of A that gives the maximum area of OABC.

7. [Maximum mark: 7]

Consider $f(x) = \log_k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.
The equation $f(x) = 2$ has exactly one solution. Find the value of k.

Do **not** write solutions on this page.

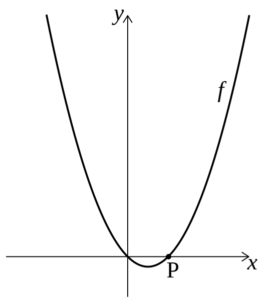
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

diagram not to scale



The graph of f crosses the x-axis at the origin and at the point $\text{P}(1, 0)$.

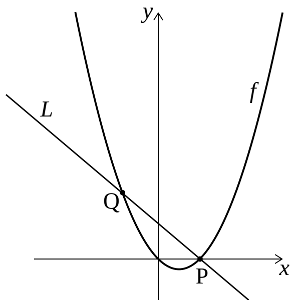
(a) Show that $f'(1) = 1$. [3]

The line L is the normal to the graph of f at P.

(b) Find the equation of L in the form $y = ax + b$. [3]

The line L intersects the graph of f at another point Q, as shown in the following diagram.

diagram not to scale



(c) Find the x-coordinate of Q. [4]

(d) Find the area of the region enclosed by the graph of f and the line L. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A line L passes through points $\text{A}(-3, 4, 2)$ and $\text{B}(-1, 3, 3)$.

(a) (i) Show that $\overrightarrow{\text{AB}} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(ii) Find a vector equation for L. [3]

The line L also passes through the point $\text{C}(3, 1, p)$.

(b) Find the value of p. [5]

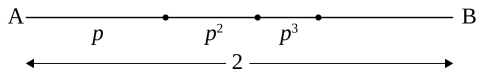
(c) The point D has coordinates $(q^2, 0, q)$. Given that $\overrightarrow{\text{DC}}$ is perpendicular to L, find the possible values of q. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

(a) The following diagram shows [AB], with length $2\,\text{cm}$. The line is divided into an infinite number of line segments. The diagram shows the first three segments.

diagram not to scale

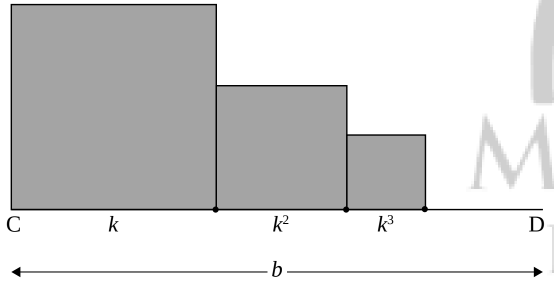


The length of the line segments are $p\text{ cm}$, $p^2\text{ cm}$, $p^3\text{ cm}$, $\ldots$, where $0 < p < 1$.

Show that $p = \frac{2}{3}$. [5]

(b) The following diagram shows [CD], with length $b\text{ cm}$, where $b > 1$. Squares with side lengths $k\text{ cm}$, $k^2\text{ cm}$, $k^3\text{ cm}$, $\ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.

diagram not to scale



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b. [9]

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Answers written on this page will not be marked.

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Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Tuesday 14 November 2017 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

15 pages

8817 – 7302

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

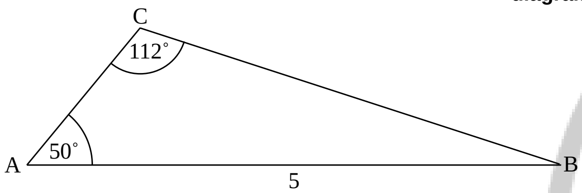
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows a triangle ABC.

diagram not to scale



$\mathrm{AB} = 5\,\mathrm{cm}$, $\mathrm{C\hat{A}B} = 50°$ and $\mathrm{A\hat{C}B} = 112°$

(a) Find BC. [3]

(b) Find the area of triangle ABC. [3]

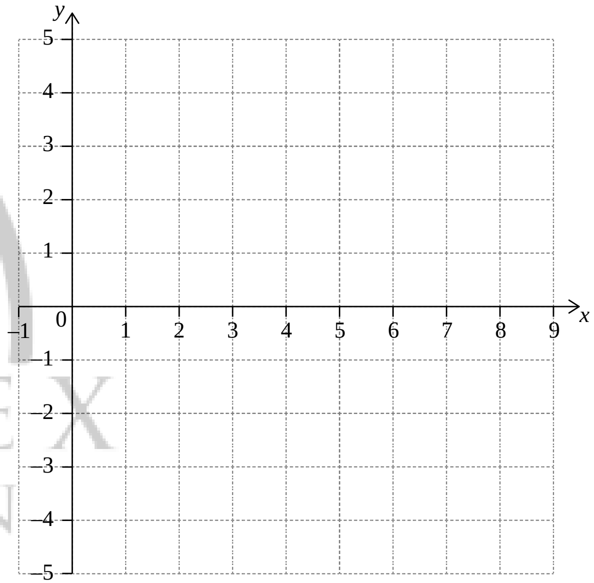
2. [Maximum mark: 7]

Let $f(x) = \dfrac{6x^2 - 4}{e^x}$, for $0 \le x \le 7$.

(a) Find the x-intercept of the graph of f. [2]

(b) The graph of f has a maximum at the point A. Write down the coordinates of A. [2]

(c) On the following grid, sketch the graph of f. [3]



3. [Maximum mark: 6]

Let $\vec{AB} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

(a) Find $\left|\vec{AB}\right|$. [2]

(b) Let $\vec{AC} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $B\hat{A}C$. [4]

4. [Maximum mark: 8]

A discrete random variable X has the following probability distribution.

X	0	1	2	3
$P(X = x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

(a) Find the value of k. [4]

(b) Write down $P(X = 2)$. [1]

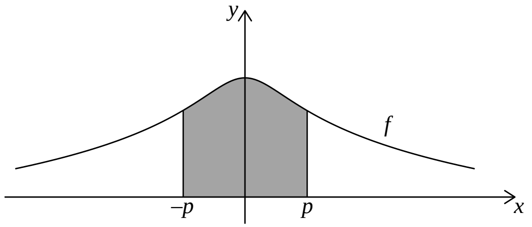
(c) Find $P(X = 2 \mid X > 0)$. [3]

5. [Maximum mark: 5]

Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.

(a) Find the value of p. [2]

(b) The following diagram shows part of the graph of f.



The region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$ is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

6. [Maximum mark: 6]

In the expansion of $ax^3(2 + ax)^{11}$, the coefficient of the term in x^5 is 11880. Find the value of a.

7. [Maximum mark: 7]

The heights of adult males in a country are normally distributed with a mean of $180\,\text{cm}$ and a standard deviation of $\sigma\,\text{cm}$. $17\,\%$ of these men are shorter than $168\,\text{cm}$. $80\,\%$ of them have heights between $(192-h)\,\text{cm}$ and $192\,\text{cm}$.

Find the value of h.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

Adam is a beekeeper who collected data about monthly honey production in his bee hives. The data for six of his hives is shown in the following table.

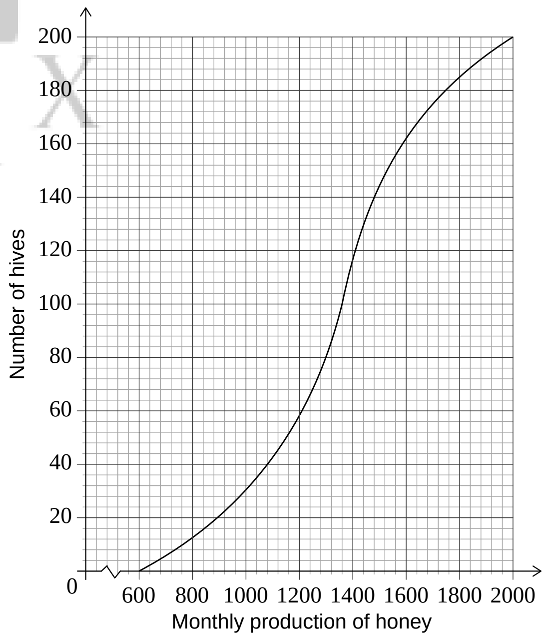
Number of bees (N)	190	220	250	285	305	320
Monthly honey production in grams (P)	900	1100	1200	1500	1700	1800

The relationship between the variables is modelled by the regression line with equation $P = aN + b$.

(a) Write down the value of a and of b. [3]

(b) Use this regression line to estimate the monthly honey production from a hive that has 270 bees. [2]

Adam has 200 hives in total. He collects data on the monthly honey production of all the hives. This data is shown in the following cumulative frequency graph.



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(Question 8 continued)

Adam's hives are labelled as low, regular or high production, as defined in the following table.

Type of hive	low	regular	high
Monthly honey production in grams (P)	$P \le 1080$	$1080 < P \le k$	$P > k$

(c) Write down the number of low production hives. [1]

Adam knows that 128 of his hives have a regular production.

(d) Find

(i) the value of k;

(ii) the number of hives that have a high production. [5]

(e) Adam decides to increase the number of bees in each low production hive. Research suggests that there is a probability of 0.75 that a low production hive becomes a regular production hive. Calculate the probability that 30 low production hives become regular production hives. [3]

Please **do not** write on this page.

Answers written on this page will not be marked.

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Note: In this question, distance is in metres and time is in seconds.

A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a = 3t^2 - 14t + 8$, for $0 \le t \le 5$.

(a) Write down the values of t when $a = 0$. [2]

(b) Hence or otherwise, find all possible values of t for which the velocity of P is decreasing. [2]

When $t = 0$, the velocity of P is $3\,\text{m}\,\text{s}^{-1}$.

(c) Find an expression for the velocity of P at time t. [6]

(d) Find the total distance travelled by P when its velocity is increasing. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 17]

Note: In this question, distance is in millimetres.

Let $f(x) = x + a\sin\left(x - \frac{\pi}{2}\right) + a$, for $x \ge 0$.

(a) Show that $f(2\pi) = 2\pi$. [3]

The graph of f passes through the origin. Let P_k be any point on the graph of f with x-coordinate $2k\pi$, where $k \in \mathbb{N}$. A straight line L passes through all the points P_k.

(b) (i) Find the coordinates of P_0 and of P_1.

(ii) Find the equation of L. [6]

(c) Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π. [2]

(This question continues on the following page)

Turn over

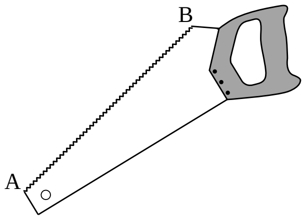
Do **not** write solutions on this page.

(Question 10 continued)

Diagram 1 shows a saw. The length of the toothed edge is the distance AB.

Diagram 1

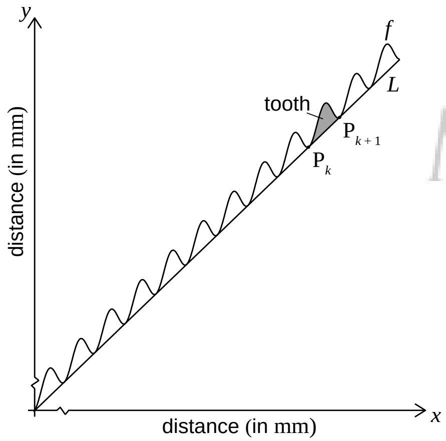
diagram not to scale



The toothed edge of the saw can be modelled using the graph of f and the line L.
Diagram 2 represents this model.

Diagram 2

diagram not to scale



The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of f and the line L, between P_k and P_{k+1}.

(d) A saw has a toothed edge which is $300\,\text{mm}$ long. Find the number of complete teeth on this saw. [6]

Please **do not** write on this page.

Answers written on this page will not be marked.

N17/5/MATME/SP2/ENG/TZ0/XX/M

Markscheme

November 2017

Mathematics

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show
a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) evidence of choosing sine rule *(M1)*

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution *(A1)*

eg $\frac{\text{BC}}{\sin 50} = \frac{5}{\sin 112}$

4.13102

$\text{BC} = 4.13$ (cm) *A1* *N2*

[3 marks]

(b) correct working *(A1)*

eg $\hat{B} = 180 - 50 - 112$, $18°$, $\text{AC} = 1.66642$

correct substitution into area formula *(A1)*

eg $\frac{1}{2} \times 5 \times 4.13 \times \sin 18$, $0.5(5)(1.66642)\sin 50$, $\frac{1}{2}(4.13)(1.66642)\sin 112$

3.19139

area $= 3.19$ (cm^2) *A1* *N2*

[3 marks]

Total [6 marks]

2. (a) valid approach *(M1)*

eg $f(x) = 0$, ± 0.816

0.816496

$x = \sqrt{\frac{2}{3}}$ (exact), 0.816 *A1* *N2*

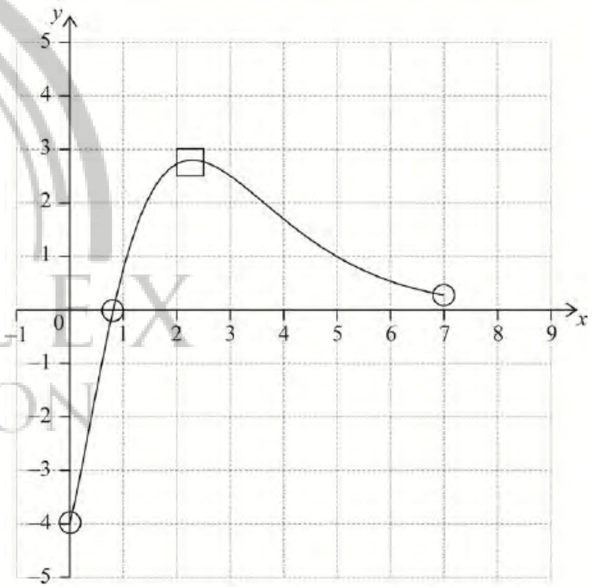
[2 marks]

(b) (2.29099, 2.78124)

$\text{A}(2.29,\ 2.78)$ *A1A1* *N2*

[2 marks]

(c)



A1A1A1 *N3*

Notes: Award ***A1*** for correct domain and endpoints at $x = 0$ and $x = 7$ in circles,
A1 for maximum in square,
A1 for approximately correct shape that passes through **their** x-intercept in circle and has changed from concave down to concave up between 2.29 and 7.

[3 marks]

Total [7 marks]

3. (a) correct substitution *(A1)*

eg $\sqrt{4^2+1^2+2^2}$

4.58257

$\left|\vec{AB}\right| = \sqrt{21}$ (exact), 4.58 ***A1*** ***N2***

[2 marks]

(b) finding scalar product and $\left|\vec{AC}\right|$ *(A1)(A1)*

scalar product $= (4\times3)+(1\times0)+(2\times0)$ $(=12)$

$\left|\vec{AC}\right| = \sqrt{3^2+0+0}$ $(=3)$

substituting **their** values into cosine formula *(M1)*

eg $\cos B\hat{A}C = \frac{4\times3+0+0}{\sqrt{3^2}\times\sqrt{21}}$, $\frac{4}{\sqrt{21}}$, $\cos\theta = 0.873$

0.509739 $(29.2059°)$

$B\hat{A}C = 0.510$ $(29.2°)$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

4. (a) valid approach *(M1)*

eg total probability = 1

correct equation *(A1)*

eg $0.475+2k^2+\frac{k}{10}+6k^2=1$, $8k^2+0.1k-0.525=0$

$k = 0.25$ ***A2*** ***N3***

[4 marks]

(b) $\mathrm{P}(X=2)=0.025$ ***A1*** ***N1***

[1 mark]

(c) valid approach for finding $\mathrm{P}(X>0)$ *(M1)*

eg $1-0.475$, $2(0.25^2)+0.025+6(0.25^2)$, $1-\mathrm{P}(X=0)$, $2k^2+\frac{k}{10}+6k^2$

correct substitution into formula for conditional probability *(A1)*

eg $\frac{0.025}{1-0.475}$, $\frac{0.025}{0.525}$

0.0476190

$\mathrm{P}(X=2\mid X>0)=\frac{1}{21}$ (exact), 0.0476 ***A1*** ***N2***

[3 marks]

Total [8 marks]

5. (a) valid approach *(M1)*

eg $f(p)=4$, intersection with $y=4$, ±2.32

2.32143

$p=\sqrt{e^2-2}$ (exact), 2.32 ***A1*** ***N2***

[2 marks]

(b) attempt to substitute **either their** limits **or** the function into volume formula (must involve f^2, accept reversed limits and absence of π and/or dx, but do not accept any other errors) *(M1)*

eg $\int_{-2.32}^{2.32} f^2$, $\pi\int\left(6-\ln(x^2+2)\right)^2 dx$, 105.675

331.989

volume = 332 ***A2*** ***N3***

[3 marks]

Total [5 marks]

6. valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for *r*) ***(M1)***

eg $\binom{11}{r}(2)^{11-r}ax^r$, $\binom{11}{3}(2)^8(ax)^3$, $2^{11}+\binom{11}{1}(2)^{10}(ax)^1+\binom{11}{2}(2)^9(ax)^2+\ldots$

recognizing need to find term in x^2 in binomial expansion ***(A1)***

eg $r=2$, $(ax)^2$

correct term or coefficient in binomial expansion (may be seen in equation) ***(A1)***

eg $\binom{11}{2}(ax)^2(2)^9$, $55(a^2x^2)(512)$, $28160a^2$

setting up equation in x^5 with **their** coefficient/term (do not accept other powers of *x*) ***(M1)***

eg $ax^3\binom{11}{2}(ax)^2(2)^9=11880x^5$

correct equation ***(A1)***

eg $28160a^3=11880$

$a=\frac{3}{4}$ ***A1 N3***

[6 marks]

7. finding the *z*-value for 0.17 ***(A1)***

eg $z=-0.95416$

setting up equation to find σ, ***(M1)***

eg $z=\frac{168-180}{\sigma}$, $-0.954=\frac{-12}{\sigma}$

$\sigma=12.5765$ ***(A1)***

EITHER (Properties of the Normal curve)

correct value (seen anywhere) ***(A1)***

eg $\mathrm{P}(X<192)=0.83$, $\mathrm{P}(X>192)=0.17$

correct working ***(A1)***

eg $\mathrm{P}(X<192-h)=0.83-0.8$, $\mathrm{P}(X<192-h)=1-0.8-0.17$, $\mathrm{P}(X>192-h)=0.8+0.17$

correct equation in *h*

eg $\frac{(192-h)-180}{12.576}=-1.88079$, $192-h=156.346$ ***(A1)***

35.6536

$h=35.7$ ***A1 N3***

OR (Trial and error using different values of *h*)

two correct probabilities whose 2 sf will round up **and** down, respectively, to 0.8 ***A2***

eg $\mathrm{P}(192-35.6<X<192)=0.799706$, $\mathrm{P}(157<X<192)=0.796284$, $\mathrm{P}(192-36<X<192)=0.801824$

$h=35.7$ ***A2***

[7 marks]

Section B

8. (a) evidence of setup *(M1)*
eg correct value for a or b
$a = 6.96103$, $b = -454.805$
$a = 6.96$, $b = -455$ (accept $6.96x - 455$) *A1A1* *N3*
[3 marks]

(b) substituting $N = 270$ into **their** equation *(M1)*
eg $6.96(270) - 455$
1424.67
$P = 1420$ (g) *A1* *N2*
[2 marks]

(c) 40 (hives) *A1* *N1*
[1 mark]

(d) (i) valid approach *(M1)*
eg $128 + 40$
168 hives have a production less than k *(A1)*
$k = 1640$ *A1* *N3*

(ii) valid approach *(M1)*
eg $200 - 168$
32 (hives) *A1* *N2*
[5 marks]

(e) recognize binomial distribution (seen anywhere) *(M1)*
eg $X \sim \mathrm{B}(n,\ p)$, $\binom{n}{r} p^r (1-p)^{n-r}$
correct values *(A1)*
eg $n = 40$ (check ***FT***) and $p = 0.75$ and $r = 30$, $\binom{40}{30} 0.75^{30} (1-0.75)^{10}$
0.144364
0.144 *A1* *N2*
[3 marks]

Total [14 marks]

9. (a) $t = \frac{2}{3}$ (exact), 0.667, $t = 4$ *A1A1* *N2*
[2 marks]

(b) recognizing that v is decreasing when a is negative *(M1)*
eg $a < 0$, $3t^2 - 14t + 8 \le 0$, sketch of a
correct interval *A1* *N2*
eg $\frac{2}{3} < t < 4$
[2 marks]

(c) valid approach (do not accept a definite integral) *(M1)*
eg $v = \int a$
correct integration (accept missing c) *(A1)(A1)(A1)*
$t^3 - 7t^2 + 8t + c$
substituting $t = 0$, $v = 3$ (must have c) *(M1)*
eg $3 = 0^3 - 7(0^2) + 8(0) + c$, $c = 3$
$v = t^3 - 7t^2 + 8t + 3$ *A1* *N6*
[6 marks]

(d) recognizing that v increases outside the interval found in part (b) *(M1)*
eg $0 < t < \frac{2}{3}$, $4 < t < 5$, diagram
one correct substitution into distance formula *(A1)*
eg $\int_0^{\frac{2}{3}} |v|$, $\int_4^5 |v|$, $\int_{\frac{2}{3}}^4 |v|$, $\int_0^5 |v|$
one correct pair *(A1)*
eg 3.13580 and 11.0833, 20.9906 and 35.2097
14.2191 *A1* *N2*
$d = 14.2$ (m)
[4 marks]
Total [14 marks]

10. (a) substituting $x = 2\pi$ ***M1***

eg $2\pi + a\sin\left(2\pi - \frac{\pi}{2}\right) + a$

$2\pi + a\sin\left(\frac{3\pi}{2}\right) + a$ ***(A1)***

$2\pi - a + a$ ***A1***

$f(2\pi) = 2\pi$ ***AG*** ***N0***

[3 marks]

(b) (i) substituting the value of k ***(M1)***

$P_0(0, 0)$, $P_1(2\pi, 2\pi)$ ***A1A1*** ***N3***

(ii) attempt to find the gradient ***(M1)***

eg $\frac{2\pi - 0}{2\pi - 0}$, $m = 1$

correct working ***(A1)***

eg $\frac{y - 2\pi}{x - 2\pi} = 1$, $b = 0$, $y - 0 = 1(x - 0)$

$y = x$ ***A1*** ***N3***

[6 marks]

(c) subtracting x-coordinates of P_{k+1} and P_k (in any order) ***(M1)***

eg $2(k+1)\pi - 2k\pi$, $2k\pi - 2k\pi - 2\pi$

correct working (must be in correct order) ***A1***

eg $2k\pi + 2\pi - 2k\pi$, $\left|2k\pi - 2(k+1)\pi\right|$

distance is 2π ***AG*** ***N0***

[2 marks]

continued…

Question 10 continued

(d) **METHOD 1**

recognizing the toothed-edge as the hypotenuse ***(M1)***

eg $300^2 = x^2 + y^2$, sketch

correct working (using their equation of L) ***(A1)***

eg $300^2 = x^2 + x^2$

$x = \frac{300}{\sqrt{2}}$ (exact), 212.132 ***(A1)***

dividing their value of x by 2π $\left(\text{do not accept } \frac{300}{2\pi}\right)$ ***(M1)***

eg $\frac{212.132}{2\pi}$

33.7618 ***(A1)***

33 (teeth) ***A1*** ***N2***

METHOD 2

vertical distance of a tooth is 2π (may be seen anywhere) ***(A1)***

attempt to find the hypotenuse for one tooth ***(M1)***

eg $x^2 = (2\pi)^2 + (2\pi)^2$

$x = \sqrt{8\pi^2}$ (exact), 8.88576 ***(A1)***

dividing 300 by their value of x ***(M1)***

eg

33.7618 ***(A1)***

33 (teeth) ***A1*** ***N2***

[6 marks]

Total [17 marks]