

SINGAPORE'S
#1 HOME TUITION AGENCY

.....○

Need A Home Tutor?

🌐 singaporetuitonteachers.com

☎ +65 9695 3522

Contact Us Today For A 100% Free Tutor Request!

○.....

OUR TEST PAPERS ARE:

- ✓ **COMPLETELY FREE!**
- ✓ **SOURCED FROM TOP SCHOOLS**
- ✓ **HIGH-QUALITY**
- ✓ **USED BY 10,000+ SATISFIED STUDENTS**



SINGAPORE'S #1 HOME TUITION AGENCY

Need A Home Tutor?

 singaporetuitionteachers.com

 +65 9695 3522

Contact Us Today For A 100% Free Tutor Request!

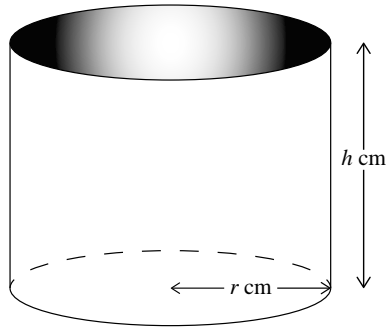
OUR TEST PAPERS ARE:

- ✓ **COMPLETELY FREE!**
- ✓ **SOURCED FROM TOP SCHOOLS**
- ✓ **HIGH-QUALITY**
- ✓ **USED BY 10,000+ SATISFIED STUDENTS**



6. [Maximum mark: 17]

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

(a) Write down a formula for A , the surface area to be coated. [2]

The volume of the water container is 0.5 m^3 .

(b) Express this volume in cm^3 . [1]

(c) Write down, in terms of r and h , an equation for the volume of this water container. [1]

(d) Show that $A = \pi r^2 + \frac{1000000}{r}$. [2]

The water container is designed so that the area to be coated is minimized.

(e) Find $\frac{dA}{dr}$. [3]

(f) Using your answer to part (e), find the value of r which minimizes A . [3]

(g) Find the value of this minimum area. [2]

One can of water-resistant material coats a surface area of 2000 cm^2 .

(h) Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

Mathematical studies Standard level Paper 1

Monday 13 November 2017 (afternoon)

1 hour 30 minutes

Candidate session number

Grid for candidate session number

Instructions to candidates

- Write your session number in the boxes above. Do not open this examination paper until instructed to do so. A graphic display calculator is required for this paper. A clean copy of the mathematical studies SL formula booklet is required for this paper. Answer all questions. Answers must be written within the answer boxes provided. Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures. The maximum mark for this examination paper is [90 marks].



Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. A group of 20 students travelled to a gymnastics tournament together. Their ages, in years, are given in the following table.

Age (years)	14	15	16	17	18	19	20	22
Frequency	1	2	7	1	4	1	1	3

- (a) For the students in this group

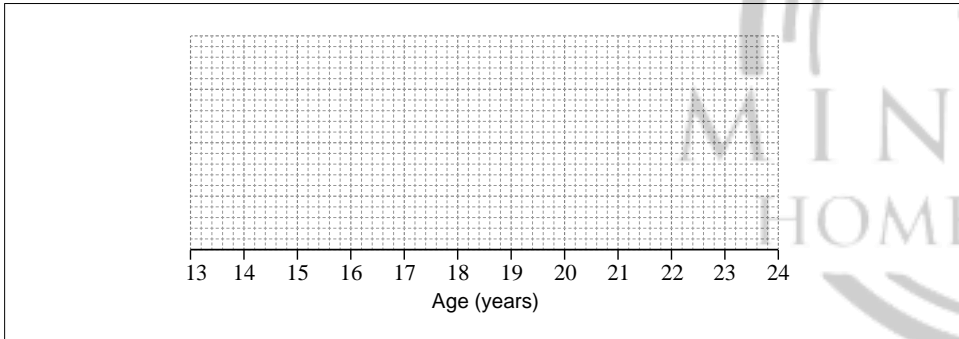
- (i) find the mean age;
- (ii) write down the median age.

[3]

The lower quartile of the ages is 16 and the upper quartile is 18.5.

- (b) Draw a box-and-whisker diagram, for these students' ages, on the following grid.

[3]



(This question continues on the following page)

(Question 1 continued)

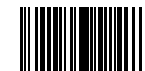
Working:

Answers:

- (a) (i)
- (ii)



20EP02



20EP03

2. The coordinates of point A are $(6, -7)$ and the coordinates of point B are $(-6, 2)$. Point M is the midpoint of AB.
- (a) Find the coordinates of M. [2]
- L_1 is the line through A and B.
- (b) Find the gradient of L_1 . [2]
- The line L_2 is perpendicular to L_1 and passes through M.
- (c) (i) Write down the gradient of L_2 .
- (ii) Write down, in the form $y = mx + c$, the equation of L_2 . [2]

Working:

Answers:

- (a)
- (b)
- (c) (i)
- (ii)

3. The speed of light is 300 000 kilometres per second. The average distance from the Sun to the Earth is 149.6 million km.
- (a) Calculate the time, **in minutes**, it takes for light from the Sun to reach the Earth. [3]
- A light-year is the distance light travels in one year and is equal to 9467 280 million km. Polaris is a bright star, visible from the Northern Hemisphere. The distance from the Earth to Polaris is 323 light-years.
- (b) Find the distance from the Earth to Polaris in millions of km. Give your answer in the form $a \times 10^k$ with $1 \leq a < 10$ and $k \in \mathbb{Z}$. [3]

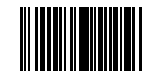
Working:

Answers:

- (a)
- (b)



20EP04



20EP05

4. Consider the following propositions.

- p : The car is under warranty
- q : The car is less than 2 years old
- r : The car has been driven more than 20000 km

- (a) Write down in words $(q \vee \neg r) \Rightarrow p$. [3]
- (b) Complete the truth table. [2]

p	q	r	$\neg r$	$q \vee \neg r$	$(q \vee \neg r) \Rightarrow p$
T	T	T	F		
T	T	F	T		
T	F	T	F		
T	F	F	T		
F	T	T	F		
F	T	F	T		
F	F	T	F		
F	F	F	T		

- (c) State whether the statement $\neg p \Rightarrow \neg(q \vee \neg r)$ is the inverse, the converse or the contrapositive of the statement in part (a). [1]

Please **do not** write on this page.
Answers written on this page will not be marked.

Working:

Answers:

(a)

.....

.....

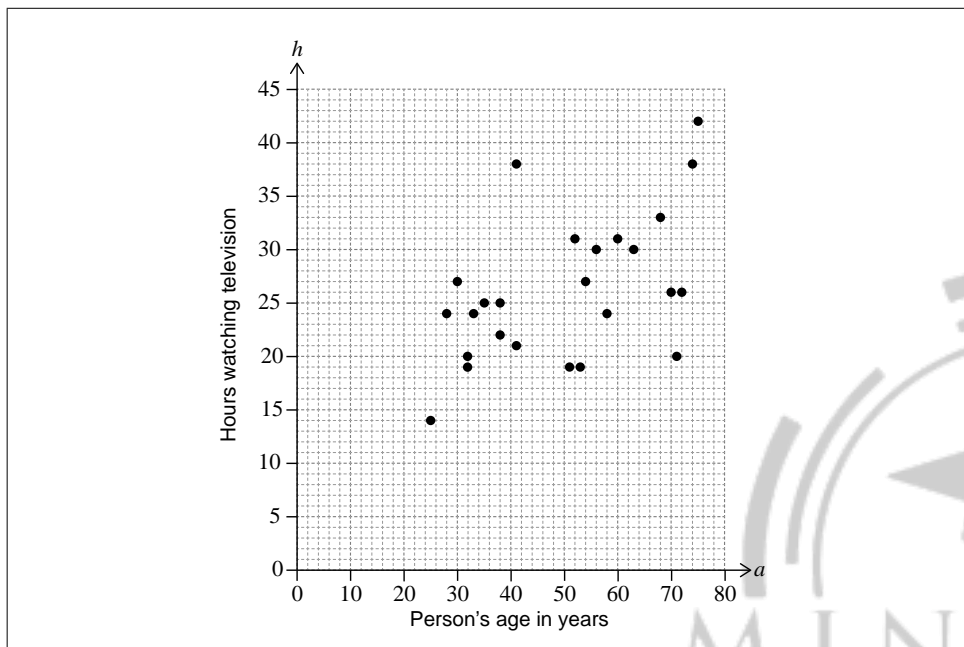
.....

.....

(c)



5. A survey was carried out to investigate the relationship between a person's age in years (a) and the number of hours they watch television per week (h). The scatter diagram represents the results of the survey.



The mean age of the people surveyed was 50.

For these results, the equation of the regression line h on a is $h = 0.22a + 15$.

- (a) Find the mean number of hours that the people surveyed watch television per week. [2]
- (b) Draw the regression line on the scatter diagram. [2]
- (c) By placing a tick (✓) in the correct box, determine which of the following statements is true:

The correlation between h and a is positive.	<input type="checkbox"/>
The correlation between h and a is negative.	<input type="checkbox"/>
There is no correlation between h and a .	<input type="checkbox"/>

[1]

- (d) Diogo is 18 years old. Give a reason why the regression line should not be used to estimate the number of hours Diogo watches television per week. [1]

(This question continues on the following page)



20EP08



20EP09

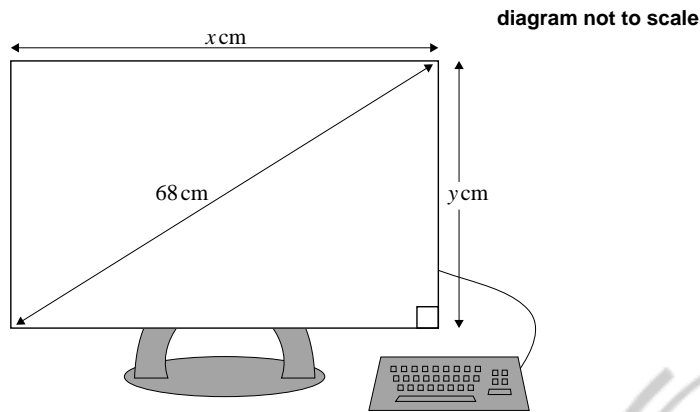
(Question 5 continued)

Working:

Answers:

- (a)
- (d)
-

6. The size of a computer screen is the length of its diagonal. Zuzana buys a rectangular computer screen with a size of 68 cm, a height of y cm and a width of x cm, as shown in the diagram.



- (a) Use this information to write down an equation involving x and y . [1]
- The ratio between the height and the width of the screen is 3:4.
- (b) Use this ratio to write down y in terms of x . [2]
- (c) Find the value of x and of y . [3]

Working:

Answers:

- (a)
- (b)
- (c)



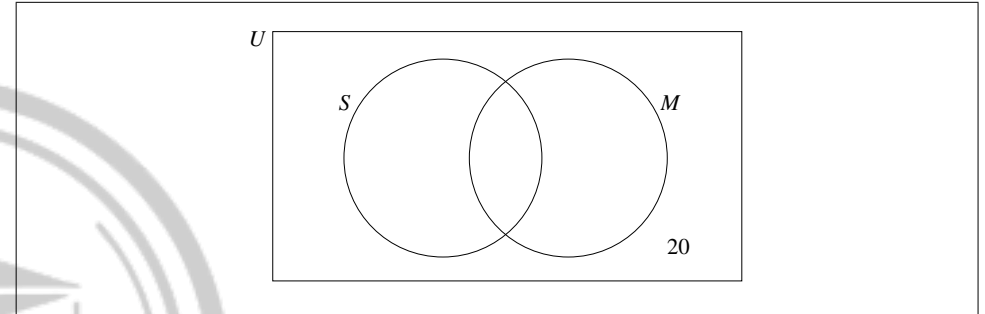
20EP10

7. Rosewood College has 120 students. The students can join the sports club (S) and the music club (M).

For a student chosen at random from these 120, the probability that they joined both clubs is $\frac{1}{4}$ and the probability that they joined the music club is $\frac{1}{3}$.

There are 20 students that did not join either club.

- (a) Complete the Venn diagram for these students. [2]



- (b) One of the students who joined the sports club is chosen at random. Find the probability that this student joined both clubs. [2]
- (c) Determine whether the events S and M are independent. [2]

Working:

Answers:

- (b)
- (c)



20EP11

8. In this question, give all answers correct to 2 decimal places.

Jose travelled from Buenos Aires to Sydney. He used Argentine pesos, ARS, to buy 350 Australian dollars, AUD, at a bank. The exchange rate was 1ARS = 0.1559AUD.

- (a) Use this exchange rate to calculate the amount of ARS that is equal to 350AUD. [2]

The bank charged Jose a commission of 2%.

- (b) Calculate the **total** amount of ARS Jose paid to get 350AUD. [2]

Jose used his credit card to pay his hotel bill in Sydney. The bill was 585AUD. The value the credit card company charged for this payment was 4228.38ARS. The exchange rate used by the credit card company was 1AUD = x ARS. No commission was charged.

- (c) Find the value of x . [2]

Working:

Answers:

- (a)
(b)
(c)



20EP12

9. Juan buys a bicycle in a sale. He gets a discount of 30% off the original price and pays 560 US dollars (USD).

- (a) Calculate the original price of the bicycle. [2]

To buy the bicycle, Juan takes a loan of 560 USD for 6 months at a nominal annual interest rate of 75%, **compounded monthly**. Juan believes that the total amount he will pay will be less than the original price of the bicycle.

- (b) Calculate the difference between the original price of the bicycle and the total amount Juan will pay. [4]

Working:

Answers:

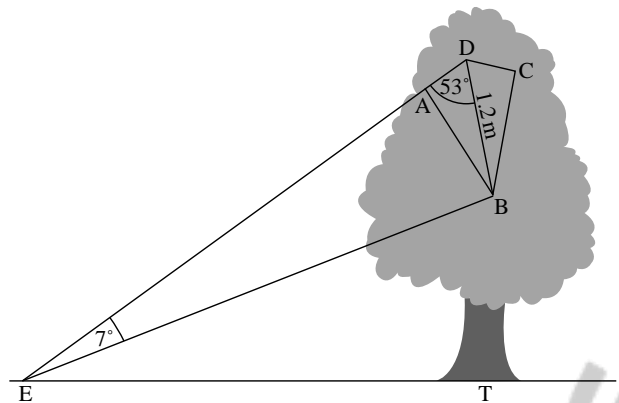
- (a)
(b)



20EP13

10. Emily's kite ABCD is hanging in a tree. The plane ABCDE is vertical.

Emily stands at point E at some distance from the tree, such that EAD is a straight line and angle $BED = 7^\circ$. Emily knows $BD = 1.2$ metres and angle $BDA = 53^\circ$, as shown in the diagram.



(a) Find the length of EB.

[3]

T is a point at the base of the tree. ET is a horizontal line. The angle of elevation of A from E is 41° .

(b) Write down the angle of elevation of B from E.

[1]

(c) Find the vertical height of B above the ground.

[2]

(This question continues on the following page)

(Question 10 continued)

Working:

Answers:

- (a)
- (b)
- (c)



20EP14



20EP15

11. A quadratic function f is given by $f(x) = ax^2 + bx + c$. The points $(0, 5)$ and $(-4, 5)$ lie on the graph of $y = f(x)$.

(a) Find the equation of the axis of symmetry of the graph of $y = f(x)$. [2]

(b) Write down the value of c . [1]

The y -coordinate of the minimum of the graph is 3.

(c) Find the value of a and of b . [3]

Working:

Answers:

- (a)
- (b)
- (c)

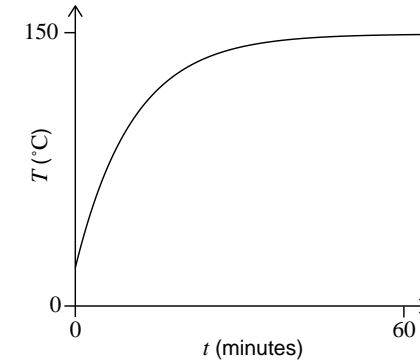


20EP16

12. Sejah placed a baking tin, that contained cake mix, in a preheated oven in order to bake a cake. The temperature in the centre of the cake mix, T , in degrees Celsius ($^{\circ}\text{C}$) is given by

$$T(t) = 150 - a \times (1.1)^{-t}$$

where t is the time, in minutes, since the baking tin was placed in the oven. The graph of T is shown in the following diagram.



(a) Write down what the value of 150 represents in the context of the question. [1]

The temperature in the centre of the cake mix was 18°C when placed in the oven.

(b) Find the value of a . [2]

The baking tin is removed from the oven 15 minutes after the temperature in the centre of the cake mix has reached 130°C .

(c) Find the total time that the baking tin is in the oven. [3]

Working:

Answers:

- (a)
-
- (b)
- (c)



20EP17

13. Applicants for a job had to complete a mathematics test. The time they took to complete the test is normally distributed with a mean of 53 minutes and a standard deviation of 16.3. One of the applicants is chosen at random.

(a) Find the probability that this applicant took at least 40 minutes to complete the test. [2]

For 11 % of the applicants it took longer than k minutes to complete the test.

(b) Find the value of k . [2]

There were 400 applicants for the job.

(c) Estimate the number of applicants who completed the test in less than 25 minutes. [2]

Working:

Answers:

- (a)
- (b)
- (c)



20EP18

14. A function f is given by $f(x) = 4x^3 + \frac{3}{x^2} - 3, x \neq 0$.

(a) Write down the derivative of f . [3]

(b) Find the point on the graph of f at which the gradient of the tangent is equal to 6. [3]

Working:

Answers:

- (a)
- (b)



20EP19



15. Maria owns a cheese factory. The amount of cheese, in kilograms, Maria sells in one week, Q , is given by

$$Q = 882 - 45p,$$

where p is the price of a kilogram of cheese in euros (EUR).

- (a) Write down how many kilograms of cheese Maria sells in one week if the price of a kilogram of cheese is 8 EUR. [1]

Maria earns $(p - 6.80)$ EUR for each kilogram of cheese sold.

- (b) Find how much Maria earns in one week, from selling cheese, if the price of a kilogram of cheese is 8 EUR. [2]

To calculate her weekly profit W , in EUR, Maria multiplies the amount of cheese she sells by the amount she earns per kilogram.

- (c) Write down an expression for W in terms of p . [1]
- (d) Find the price, p , that will give Maria the highest weekly profit. [2]

Working:

Answers:

- (a)
- (b)
- (c)
- (d)

Markscheme

November 2017

Mathematical studies

Standard level

Paper 1



**Paper 1 Markscheme
Instructions to Examiners**

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6.

1 Abbreviations

The markscheme may make use of the following abbreviations:

- M** Marks awarded for **Method**
- A** Marks awarded for an **Answer** or for **Accuracy**
- C** Marks awarded for **Correct** answers (irrespective of working shown)
- R** Marks awarded for clear **Reasoning**
- ft** Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

- (a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If the candidate has full marks on a question use the **C6** annotation, if the candidate has made an attempt but scores zero marks use **C0**. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.
- (c) In this paper, if the **correct answer is seen on the answer line** the maximum mark is awarded. **There is no need to check the working!** Award **C** marks and move on.
- (d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.
- (e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.
- (f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.
- (g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.
- (h) If correct working results in a correct answer **in the working box** but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final (A1) (ignore the further working)
2.	$(x - 6)(x + 1)$	$x = 6$ and -1	Do not award the final (A1) (see next example)

This markscheme is the property of the International Baccalaureate and must **not** be reproduced or distributed to any other person without the authorization of the IB Global Centre, Cardiff.

Example: Factorise $x^2 - 5x - 6$

Markscheme	Candidates' Scripts	Marking
$(x-6)(x+1)$ (A1)(A1)	(i) Answer line: $(x+6)(x+1)$	(A0)(A1)
	(ii) Working box: $(x-6)(x+1)$ followed by $x=6$ and -1 , or just $6, -1$ in either working box or on answer line.	(A1) (A0)

3 Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(ft)'.

- Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- If an answer resulting from follow through is extremely unrealistic (eg, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ (M1)(A1)	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$	(M1)(A0)
$A = 22.0^\circ$ (22.0243...) (A1)	$A = 41.8^\circ$ (Note: the 2 nd (A1) here was not marked (ft) and cannot be awarded because there was an earlier error in the same question part.)	(A0)
(b) $x = 7 \tan (22.0243\dots^\circ)$ (M1) $= 2.83$ (2.83163...) (A1)(ft)	(b) case (i) $x = 7 \tan 41.8^\circ$ $= 6.26$ but case (ii) 6.26	(M1) (A1)(ft) (C0) since no working shown

4 Using the Markscheme

- A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark. The only exception will be for an answer where the accuracy is specified in the question – see section 5.
- A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** is not awarded for a correct answer if no reason or the wrong reason is given.
- Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme. Where alternative methods for complete questions are included in the markscheme, they are indicated by 'OR' etc.
- Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$. On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value (for example $\frac{2}{3}$ if applicable);
the full calculator display in the form 2.83163... as in the example above. Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.
- As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (e.g. complement): A' ; \bar{A} ; A^c ; $U - A$; $(A; U \setminus A$.

Different forms of logic notation: $\neg p$; p' ; \bar{p} ; $\sim p$.
 $p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

Significance level may be written as α .

- Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(A1)** and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

2. If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

	If candidates final answer is given ...					
	Exact or to 4 or more sf (and would round to the correct 3 sf)	Correct to 3 sf	Incorrect to 3 sf	Correct to 2 sf ³	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen ¹	Award the final (A1) irrespective of correct or incorrect rounding					
Unrounded answer not seen ²	(A1)	(A1)	(A0)	(A1)	(A0)	(A0)
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen. ³			

Examples:

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) (A1)	(i) 9.43398... is seen in the working box followed by 9; 9.4; 9.43; 9.434 etc. (correctly rounded)	(A1)
	(ii) 9.43398... is seen in the working box followed by 9.433; 9.44 etc. (incorrectly rounded)	(A1)
	(iii) 9.4	(A1)
	(iv) 9	(A0) (correct to 1 sf)
	(v) 9.3	(A0) (incorrectly rounded to 2 sf)
	(vi) 9.44	(A0) (incorrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) (A1)	(i) 7.43798... is seen in the working box followed by 7; 7.4; 7.44; 7.438 etc. (correctly rounded)	(A1)
	(ii) 7.43798... is seen in the working box followed by 7.437; 7.43 etc. (incorrectly rounded)	(A1)
	(iii) 7.4	(A1)
	(iv) 7	(A0) (correct to 1 sf)
	(v) 7.5	(A0) (incorrectly rounded to 2 sf)
	(vi) 7.43	(A0) (incorrectly rounded to 3 sf)

Example: ABC is a right angled triangle with angle $\angle C = 90^\circ$, $AC = 32$ cm and $AB = 30$ cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ (M1) Award (M1) for correct substitution in Pythagoras' formula	(a) $BC = \sqrt{32^2 - 30^2}$ (M1) 11 (cm) (A1) <i>(2 sf answer only seen, but correct)</i>	
$= 11.1 (\sqrt{124}, 11.1355\dots)$ (cm) (A1)		
(b) Area $= \frac{1}{2} \times 30 \times 11.1355\dots$ (M1) Award (M1) for correct substitution in area of triangle formula	(b) case (i) Area $= \frac{1}{2} \times 30 \times 11$ (M1) <i>(working shown)</i>	
$= 167(167.032\dots)$ (cm ²) (A1)(ft)	$= 165$ (cm ²) (A1)(ft)	
	case (ii) $= 165$ (cm ²) (M0)(A0)(ft) <i>(No working shown, the answer 11 is treated as a ft, so no marks awarded here)</i>	

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}, 2^{\frac{3}{4}}, \sqrt[3]{5}$.) may be accepted as exact answers. All other powers (eg. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg. Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	(A2)
7.68 (7.67543...)	(b) 7.67	(A1)
	(c) 7.6	(A1)
	(d) 8	(A0)
	(e) 7	(A0)
	(e) 7.66	(A0)

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ (A2) ($y = 0.887686\dots x + 13.4895\dots$) If an answer is not in the form of an equation award at most (A1)(A0) .	(a) $y = 0.89x + 13$	(A2) <i>(both accepted)</i>
	(b) $y = 0.88x + 13$	(A1) <i>(one rounding error)</i>
	(c) $y = 0.88x + 14$	(A1) <i>(rounding error repeated)</i>
	(d) (i) $y = 0.9x + 13$ (ii) $y = 0.8x + 13$	(A1) <i>(1 sf not accepted)</i>
	(e) $0.88x + 14$	(A0) <i>(two rounding errors and not an equation)</i>

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
(2.06, 4.49) (A1)(A1) (2.06020..., 4.49253...)	(a) (2.1, 4.5)	(A1)(A1) <i>(both accepted)</i>
	(b) (2.0, 4.4)	(A1) <i>(same rounding error twice)</i>
	(c) (2.06, 4.4)	(A1) <i>(one rounding error)</i>
	(d) (2, 4.4)	(A0) <i>(1sf not accepted, one rounding error)</i>

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) (A1)	(i) 231.6	(A0)
	(ii) 232 <i>(Correct rounding to incorrect level)</i>	(A0)
	(iii) 231.61	(A0)
	(iv) 232.00 <i>(Parts (iii) and (iv) are both incorrect rounding to correct level)</i>	(A0)

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units. The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

Markscheme	Candidates' Scripts	Marking
(a) 37 000 m ² (A1)	(a) 36 000 m ² <i>(Incorrect answer so units not considered)</i>	(A0)
(b) 3 200 m ³ (A1)	(b) 3 200 m ² <i>(Incorrect units)</i>	(A0)

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

8 Graphic Display Calculators

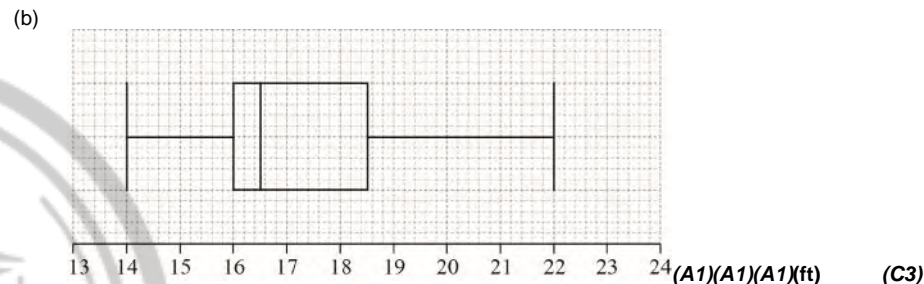
Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

1. (a) (i) $\frac{14 + 2 \times 15 + 7 \times 16 + 17 + 4 \times 18 + 19 + 20 + 3 \times 22}{20}$ **(M1)**

Note: Award **(M1)** for correct substitutions into mean formula.

(=) 17.5 **(A1) (C2)**

(ii) 16.5 **(A1) (C1)**
[3 marks]



Note: Award **(A1)** for correct endpoints, **(A1)** for correct quartiles, **(A1)(ft)** for their median. Follow through from part (a)(ii), but only if median is between 16 and 18.5. If a horizontal line goes through the box, award at most **(A1)(A1)(A0)**. Award at most **(A0)(A1)(A1)** if a ruler has not been used.

[3 marks]

Total [6 marks]

2. (a) $(0, -2.5)$ OR $\left(0, -\frac{5}{2}\right)$ (A1)(A1) (C2)

Note: Award (A1) for 0 and (A1) for -2.5 written as a coordinate pair. Award at most (A1)(A0) if brackets are missing. Accept " $x = 0$ and $y = -2.5$ ".

[2 marks]

(b) $\frac{2 - (-7)}{-6 - 6}$ (M1)

Note: Award (M1) for correct substitution into gradient formula.

$= -\frac{3}{4}$ (-0.75) (A1) (C2)

[2 marks]

(c) (i) $\frac{4}{3}$ (1.33333...) (A1)(ft) (C1)

Note: Award (A0) for $\frac{1}{0.75}$. Follow through from part (b).

(ii) $y = \frac{4}{3}x - \frac{5}{2}$ ($y = 1.33\dots x - 2.5$) (A1)(ft) (C1)

Note: Follow through from parts (c)(i) and (a). Award (A0) if final answer is not written in the form $y = mx + c$.

[2 marks]

Total [6 marks]

3. (a) $\frac{149600000}{300000 \times 60}$ (M1)(M1)

Note: Award (M1) for dividing the correct numerator (which can be presented in a different form such as 149.6×10^6 or 1.496×10^8) by 300000 and (M1) for dividing by 60.

$= 8.31$ (minutes) (8.31111..., 8 minutes 19 seconds) (A1) (C3)

[3 marks]

(b) 323×9467280 (M1)

Note: Award (M1) for multiplying 323 by 9467280, seen with any power of 10; therefore only penalizing incorrect power of 10 once.

$= 3.06 \times 10^9$ ($= 3.05793\dots \times 10^9$) (A1)(A1) (C3)

Note: Award (A1) for 3.06.
Award (A1) for $\times 10^9$.
Award (A0)(A0) for answers of the type: 30.6×10^8 .

[3 marks]

Total [6 marks]

4. (a) if the car is less than 2 years old or the car has not been driven more than 20 000 km, then the car is under warranty (A1)(A1)(A1) (C3)

Note: Award (A1) for if ..., then ..., (A1) for "or", (A1) for correct statements in correct order. Accept "If the car has not been driven more than 20 000 km or the car is less than 2 years old, then the car is under warranty". Accept logical equivalent wording for each proposition, eg "less than 20 000 km".

[3 marks]

(b)

p	q	r	$\neg r$	$q \vee \neg r$	$(q \vee \neg r) \Rightarrow p$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	T
F	F	F	T	T	F

(A1)(A1)(ft) (C2)

Note: Award (A1) for $q \vee \neg r$ column correct and (A1)(ft) for $(q \vee \neg r) \Rightarrow p$ column correct. Follow through from their $q \vee \neg r$ column.

[2 marks]

(c) contrapositive

(A1) (C1)
[1 mark]

Total [6 marks]

5. (a) $0.22(50) + 15$ (M1)

Note: Award (M1) for correct substitution of 50 into equation of the regression line.

$(=) 26$ (A1) (C2)

OR

$\frac{655}{25}$ (M1)

Note: Award (M1) for correctly summing the h values of the points, and dividing by 25.

$(=) 26.2$ (A1) (C2)
[2 marks]

- (b) line through $(50, 26 \pm 1)$ and $(0, 15)$ (A1)(ft)(A1) (C2)

Note: Award (A1)(ft) for a straight line through $(50, \text{their } \bar{h})$, and (A1) for the line intercepting the y -axis at $(0, 15)$; this may need to be extrapolated. Follow through from part (a). Award at most (A0)(A1) if the line is not drawn with a ruler.

[2 marks]

(c)

The correlation between h and a is positive.	✓
The correlation between h and a is negative.	
There is no correlation between h and a .	

(A1) (C1)

Note: Award (A0) if more than one tick (✓) is seen.

[1 mark]

- (d) 18 is less than the lowest age in the survey OR extrapolation. (A1) (C1)

Note: Accept equivalent statements.

[1 mark]

Total [6 marks]

6. (a) $x^2 + y^2 = 68^2$ (or 4624 or equivalent)

(A1) (C1)
[1 mark]

(b) $\frac{y}{x} = \frac{3}{4}$

(M1)

Note: Award (M1) for a correct equation.

$y = \frac{3}{4}x$ ($y = 0.75x$)

(A1) (C2)
[2 marks]

(c) $x^2 + \left(\frac{3}{4}x\right)^2 = 68^2$ (or $x^2 + \frac{9}{16}x^2 = 4624$ or equivalent)

(M1)

Note: Award (M1) for correct substitution of their expression for y into their answer to part (a). Accept correct substitution of x in terms of y .

$x = 54.4$ (cm), $y = 40.8$ (cm)

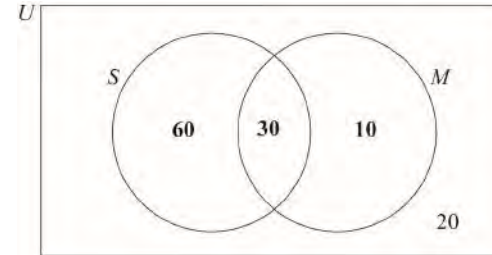
(A1)(ft)(A1)(ft) (C3)

Note: Follow through from parts (a) and (b) as long as $x > 0$ and $y > 0$.

[3 marks]

Total [6 marks]

7. (a)



(A1)(A1) (C2)

Note: Award (A1) for 30 in correct area, (A1) for 60 and 10 in the correct areas.

[2 marks]

(b) $\frac{30}{90} \left(\frac{1}{3}, 0.333333\dots, 33.3333\dots\% \right)$

(A1)(ft)(A1)(ft) (C2)

Note: Award (A1)(ft) for correct numerator of 30, (A1)(ft) for correct denominator of 90. Follow through from their Venn diagram.

[2 marks]

(c) $P(S) \times P(M) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

(R1)

Note: Award (R1) for multiplying their $P(S)$ by $\frac{1}{3}$.

therefore the events are independent (as $P(S \cap M) = \frac{1}{4}$)

(A1)(ft) (C2)

Note: Award (R1)(A1)(ft) for an answer which is consistent with their Venn diagram. Do not award (R0)(A1)(ft). Do not award final (A1) if $P(S) \times P(M)$ is not calculated. Follow through from part (a).

[2 marks]

Total [6 marks]

8. **Note:** In this question, the first time an answer is not to 2 dp the final **(A1)** is not awarded.

(a) $\frac{350}{0.1559}$ **(M1)**

Note: Award **(M1)** for dividing 350 by 0.1559.

= 2245.03 (ARS) **(A1) (C2)**
[2 marks]

(b) 2245.03×1.02 **(M1)**

Note: Award **(M1)** for multiplying their answer to part (a) by 1.02.

= 2289.93 (ARS) **(A1)(ft) (C2)**

OR

2245.03×0.02 **(M1)**

Note: Award **(M1)** for multiplying their answer to part (a) by 0.02.

= 44.9006
2245.03 + 44.90
= 2289.93 (ARS) **(A1)(ft) (C2)**

Note: Follow through from part (a).

(c) $\frac{4228.38}{585}$ **(M1)**

Note: Award **(M1)** for dividing 4228.38 by 585.

= 7.23 **(A1) (C2)**
[2 marks]

Total [6 marks]

9. (a) $\frac{560}{70} \times 100$ (or equivalent) **(M1)**

Note: Award **(M1)** for dividing 560 by 0.7 or equivalent.

= 800 (USD) **(A1) (C2)**
[2 marks]

(b) $560 \left(1 + \frac{75}{12 \times 100}\right)^{12 \times \frac{1}{2}}$ **(M1)(A1)**

Note: Award **(M1)** for substitution into interest formula, **(A1)** for their correct substitution.

OR

$N = \frac{1}{2}$
 $I\% = 75$
 $PV = (\pm)560$
 $P/Y = 1$
 $C/Y = 12$ **(A1)(M1)**

Note: Award **(A1)** for $C/Y = 12$ seen, **(M1)** for all other entries correct.

OR

$N = 6$
 $I\% = 75$
 $PV = (\pm)560$
 $P/Y = 12$
 $C/Y = 12$ **(A1)(M1)**

Note: Award **(A1)** for $C/Y = 12$ seen, **(M1)** for all other entries correct.

= 805.678... (USD) **(A1)**

Note: Award **(A3)** for 805.678... (806) seen without working.

(Juan spends) 5.68 (USD) (5.67828... USD) (more than the original price) **(A1)(ft) (C4)**

[4 marks]

Total [6 marks]

10. Units are required in parts (a) and (c).

(a) $\frac{EB}{\sin 53^\circ} = \frac{1.2}{\sin 7^\circ}$

(M1)(A1)

Note: Award (M1) for substitution into sine formula, (A1) for correct substitution.

(EB \Rightarrow) 7.86m OR 786cm (7.86385...m OR 786.385...cm)

(A1) (C3)

[3 marks]

(b) 34°

(A1) (C1)

[1 mark]

(c) $\sin 34^\circ = \frac{\text{height}}{7.86385\dots}$

(M1)

Note: Award (M1) for correct substitution into a trigonometric ratio.

(height \Rightarrow) 4.40m OR 440cm (4.39741...m OR 439.741...cm)

(A1)(ft) (C2)

Note: Accept "BT" used for height. Follow through from parts (a) and (b). Use of 7.86 gives an answer of 4.39525....

[2 marks]

Total [6 marks]

11. (a) $x = -2$

(A1)(A1) (C2)

Note: Award (A1) for $x =$ (a constant) and (A1) for -2 .

[2 marks]

(b) $(c =) 5$

(A1) (C1)

[1 mark]

(c) $-\frac{b}{2a} = -2$

$a(-2)^2 - 2b + 5 = 3$ or equivalent

$a(-4)^2 - 4b + 5 = 5$ or equivalent

$2a(-2) + b = 0$ or equivalent

(M1)

Note: Award (M1) for two of the above equations.

$a = 0.5$

(A1)(ft)

$b = 2$

(A1)(ft) (C3)

Note: Award at most (M1)(A1)(ft)(A0) if the answers are reversed. Follow through from parts (a) and (b).

[3 marks]

Total [6 marks]

MIND FLEX
HOME TUITION

12. (a) the temperature in the oven

(A1)

OR

the maximum possible temperature of the cake mix

(A1) (C1)

Note: Award (A0) for "the maximum temperature".

[1 mark]

(b) $18 = 150 - a(\times 1.1^0)$

(M1)

Note: Award (M1) for correct substitution of 18 and 0. Substitution of 0 can be implied.

(a) = 132

(A1) (C2)

[2 marks]

(c) $150 - 132 \times 1.1^{-t} = 130$

(M1)

Note: Award (M1) for substituting their a and equating to 130. Accept an inequality.
Award (M1) for a sketch of the horizontal line on the graph.

$t = 19.8$ (19.7992...)

(A1)(ft)

Note: Follow through from part (b).

34.8 (minutes) (34.7992..., 34 minutes 48 seconds)

(A1)(ft) (C3)

Note: Award the final (A1) for adding 15 minutes to their t value.
In part (c), award (C2) for a final answer of 19.8 with no working.

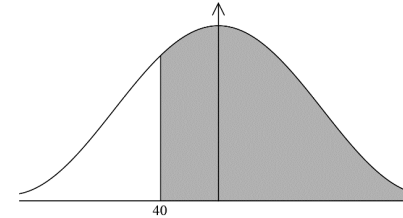
[3 marks]

Total [6 marks]

13. (a) 0.787 (0.787433..., 78.7%)

(M1)(A1) (C2)

Note: Award (M1) for a correct probability statement, $P(X > 40)$, or a correctly shaded normal distribution graph.

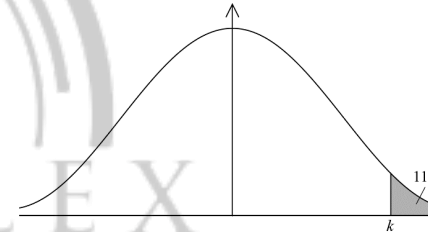


[2 marks]

(b) 73.0 (minutes) (72.9924...)

(M1)(A1) (C2)

Note: Award (M1) for a correct probability statement, $P(X > k) = 0.11$, or a correctly shaded normal distribution graph.



[2 marks]

(c) $0.0423433... \times 400$

(M1)

Note: Award (M1) for multiplying a probability by 400. Do not award (M1) for 0.11×400 .
Use of a lower bound less than zero gives a probability of 0.0429172....

= 16

(A1) (C2)

Notes: Accept a final answer of 17. Do not accept a final answer of 18. Accept a non-integer final answer either 16.9 (16.9373...) from use of lower bound zero or 17.2 (17.1669...) from use of the default lower bound of -10^{99} .

[2 marks]

Total [6 marks]

14. (a) $12x^2 - \frac{6}{x^3}$ or equivalent

(A1)(A1)(A1) (C3)

Note: Award (A1) for $12x^2$, (A1) for -6 and (A1) for $\frac{1}{x^3}$ or x^{-3} . Award at most (A1)(A1)(A0) if additional terms seen.

[3 marks]

(b) $12x^2 - \frac{6}{x^3} = 6$

(M1)

Note: Award (M1) for equating their derivative to 6.

(1, 4) OR $x = 1, y = 4$

(A1)(ft)(A1)(ft) (C3)

Note: A frequent wrong answer seen in scripts is (1, 6) for this answer with correct working award (M1)(A0)(A1) and if there is no working award (C1).

[3 marks]

Total [6 marks]

15. (a) 522 (kg)

(A1) (C1)
[1 mark]

(b) $522(8 - 6.80)$ or equivalent

(M1)

Note: Award (M1) for multiplying their answer to part (a) by $(8 - 6.80)$.

626 (EUR) (626.40)

(A1)(ft) (C2)

Note: Follow through from part (a).

[2 marks]

(c) $(W =) (882 - 45p)(p - 6.80)$

(A1)

OR

$(W =) -45p^2 + 1188p - 5997.6$

(A1) (C1)
[1 mark]

(d) sketch of W with some indication of the maximum

(M1)

OR

$-90p + 1188 = 0$

(M1)

Note: Award (M1) for equating the correct derivative of their part (c) to zero.

OR

$(p =) \frac{-1188}{2 \times (-45)}$

(M1)

Note: Award (M1) for correct substitution into the formula for axis of symmetry.

$(p =) 13.2$ (EUR)

(A1)(ft) (C2)

Note: Follow through from their part (c), if the value of p is such that $6.80 < p < 19.6$.

[2 marks]

Total [6 marks]



Mathematical studies
Standard level
Paper 2

Tuesday 14 November 2017 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 15]

A group of 800 students answered 40 questions on a category of their choice out of History, Science and Literature.

For each student the category and the number of correct answers, N , was recorded. The results obtained are represented in the following table.

		Number of correct answers, N				Total number of students
		$1 \leq N \leq 10$	$11 \leq N \leq 20$	$21 \leq N \leq 30$	$31 \leq N \leq 40$	
Category	History	46	80	68	39	233
	Science	37	82	85	56	260
	Literature	31	110	104	62	307
Total number of students		114	272	257	157	800

(a) State whether N is a discrete or a continuous variable. [1]

(b) Write down, for N ,
(i) the modal class;
(ii) the mid-interval value of the modal class. [2]

(c) Use your graphic display calculator to estimate
(i) the mean of N ;
(ii) the standard deviation of N . [3]

A χ^2 test at the 5% significance level is carried out on the results. The critical value for this test is 12.592.

(d) Find the expected frequency of students choosing the Science category and obtaining 31 to 40 correct answers. [2]

(This question continues on the following page)

(Question 1 continued)

- (e) Write down
- (i) the null hypothesis for this test;
 - (ii) the number of degrees of freedom. [2]
- (f) Write down
- (i) the p -value for the test;
 - (ii) the χ^2 statistic. [3]
- (g) State the result of the test. Give a reason for your answer. [2]

2. [Maximum mark: 15]

Rosa joins a club to prepare to run a marathon. During the first training session Rosa runs a distance of 3000 metres. Each training session she increases the distance she runs by 400 metres.

- (a) Write down the distance Rosa runs
- (i) in the third training session;
 - (ii) in the n th training session. [3]

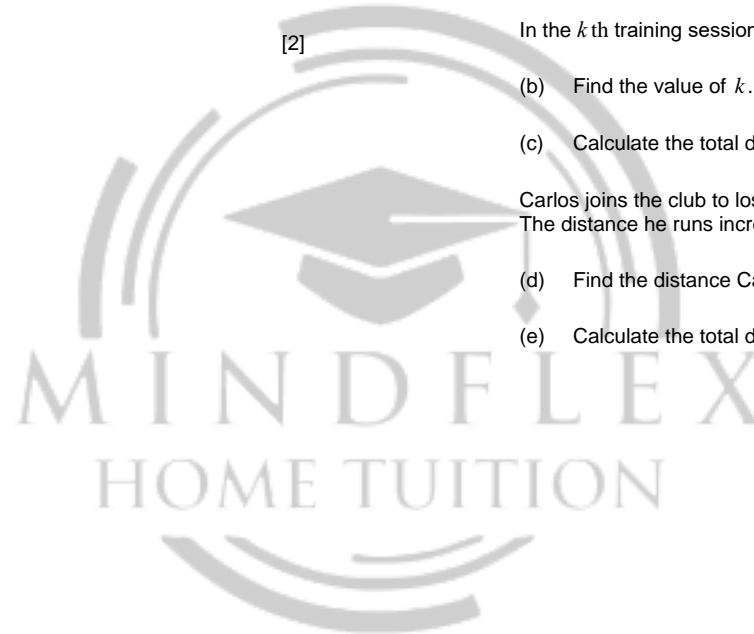
A marathon is 42.195 kilometres.

In the k th training session Rosa will run further than a marathon for the first time.

- (b) Find the value of k . [2]
- (c) Calculate the total distance, in **kilometres**, Rosa runs in the first 50 training sessions. [4]

Carlos joins the club to lose weight. He runs 7500 metres during the first month. The distance he runs increases by 20% each **month**.

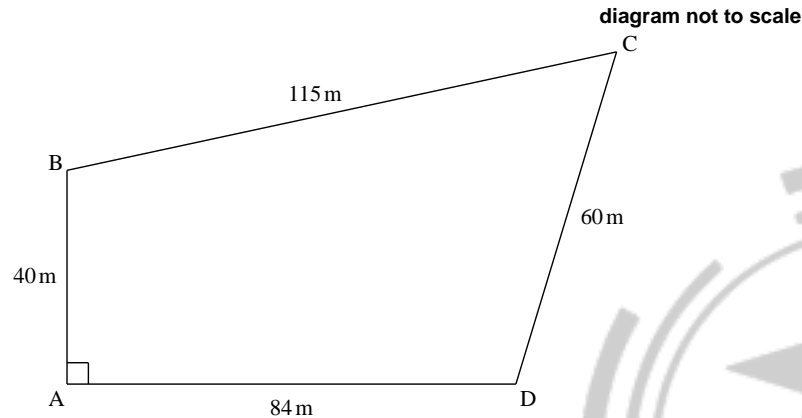
- (d) Find the distance Carlos runs in the fifth month of training. [3]
- (e) Calculate the total distance Carlos runs in the first year. [3]



3. [Maximum mark: 13]

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral ABCD. The lengths of the sides are $AB = 40\text{ m}$, $BC = 115\text{ m}$, $CD = 60\text{ m}$, $AD = 84\text{ m}$ and angle $\hat{BAD} = 90^\circ$.

This information is shown on the diagram.



- (a) Show that $BD = 93\text{ m}$ correct to the nearest metre. [2]
 (b) Calculate angle \hat{BCD} . [3]
 (c) Find the area of ABCD. [4]

The formula that the ancient Egyptians used to estimate the area of a quadrilateral ABCD is

$$\text{area} = \frac{(AB + CD)(AD + BC)}{4}$$

Abdallah uses this formula to estimate the area of his plot of land.

- (d) (i) Calculate Abdallah's estimate for the area. [4]
 (ii) Find the percentage error in Abdallah's estimate.

4. [Maximum mark: 17]

A company performs an experiment on the efficiency of a liquid that is used to detect a nut allergy.

A group of 60 people took part in the experiment. In this group 26 are allergic to nuts. One person from the group is chosen at random.

- (a) Find the probability that this person is **not** allergic to nuts. [2]

A second person is chosen from the group.

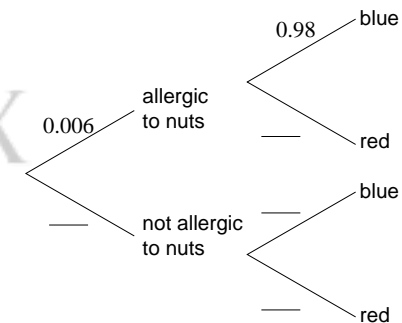
- (b) Find the probability that both people chosen are **not** allergic to nuts. [2]

When the liquid is added to a person's blood sample, it is expected to turn blue if the person is allergic to nuts and to turn red if the person is not allergic to nuts.

The company claims that the probability that the test result is correct is 98% for people who are allergic to nuts and 95% for people who are not allergic to nuts.

It is known that 6 in every 1000 adults are allergic to nuts.

This information can be represented in a tree diagram.



- (c) **Copy** and complete the tree diagram. [3]

An adult, who was not part of the original group of 60, is chosen at random and tested using this liquid.

- (d) Find the probability that this adult is allergic to nuts and the liquid turns blue. [2]
 (e) Find the probability that the liquid turns blue. [3]
 (f) Find the probability that the tested adult is allergic to nuts given that the liquid turned blue. [3]

(This question continues on the following page)

(Question 4 continued)

The liquid is used in an office to identify employees who might be allergic to nuts. The liquid turned blue for **38 employees**.

- (g) Estimate the number of employees, from this 38, who are allergic to nuts. [2]



Blank page

5. [Maximum mark: 16]

A function f is given by $f(x) = (2x + 2)(5 - x^2)$.

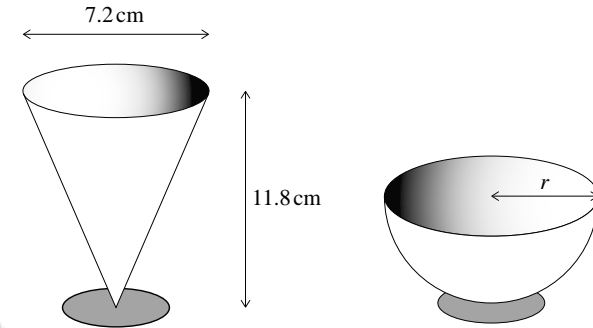
- (a) Find the **exact** value of each of the zeros of f . [3]
- (b) (i) Expand the expression for $f(x)$. [4]
- (ii) Find $f'(x)$. [4]
- (c) Use your answer to part (b)(ii) to find the values of x for which f is increasing. [3]
- (d) **Draw** the graph of f for $-3 \leq x \leq 3$ and $-40 \leq y \leq 20$. Use a scale of 2 cm to represent 1 unit on the x -axis and 1 cm to represent 5 units on the y -axis. [4]

The graph of the function $g(x) = 5^x + 6x - 6$ intersects the graph of f .

- (e) Write down the coordinates of the point of intersection. [2]

6. [Maximum mark: 14]

A restaurant serves desserts in glasses in the shape of a cone and in the shape of a hemisphere. The diameter of a cone shaped glass is 7.2 cm and the height of the cone is 11.8 cm as shown.



- (a) Show that the volume of a cone shaped glass is 160 cm^3 , correct to 3 significant figures. [2]

The volume of a hemisphere shaped glass is 225 cm^3 .

- (b) Calculate the radius, r , of a hemisphere shaped glass. [3]

The restaurant offers two types of dessert.

The **regular dessert** is a hemisphere shaped glass completely filled with chocolate mousse. The cost, to the restaurant, of the chocolate mousse for one regular dessert is \$1.89.

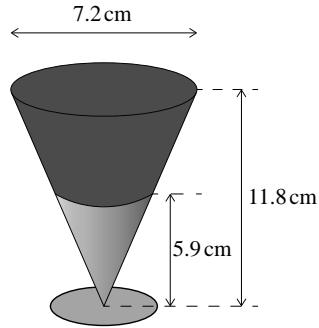
- (c) Find the cost of 100 cm^3 of chocolate mousse. [2]

(This question continues on the following page)

(Question 6 continued)

The **special dessert** is a cone shaped glass filled with two ingredients. It is first filled with orange paste to half of its height and then with chocolate mousse for the remaining volume.

diagram not to scale



- (d) Show that there is 20 cm^3 of orange paste in each special dessert.

[2]

The cost, to the restaurant, of 100 cm^3 of orange paste is \$7.42.

- (e) Find the total cost of the ingredients of one special dessert.

[2]

A chef at the restaurant prepares 50 desserts; x regular desserts and y special desserts. The cost of the ingredients for the 50 desserts is \$111.44.

- (f) Find the value of x .

[3]