Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3081135600

ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **October/November 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (PQ/FC) 187997/3

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

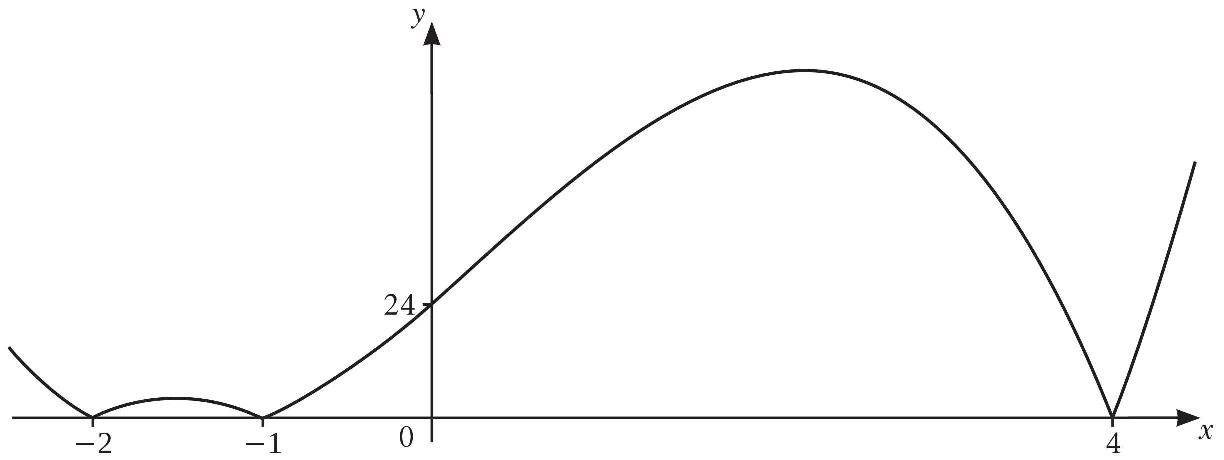
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1

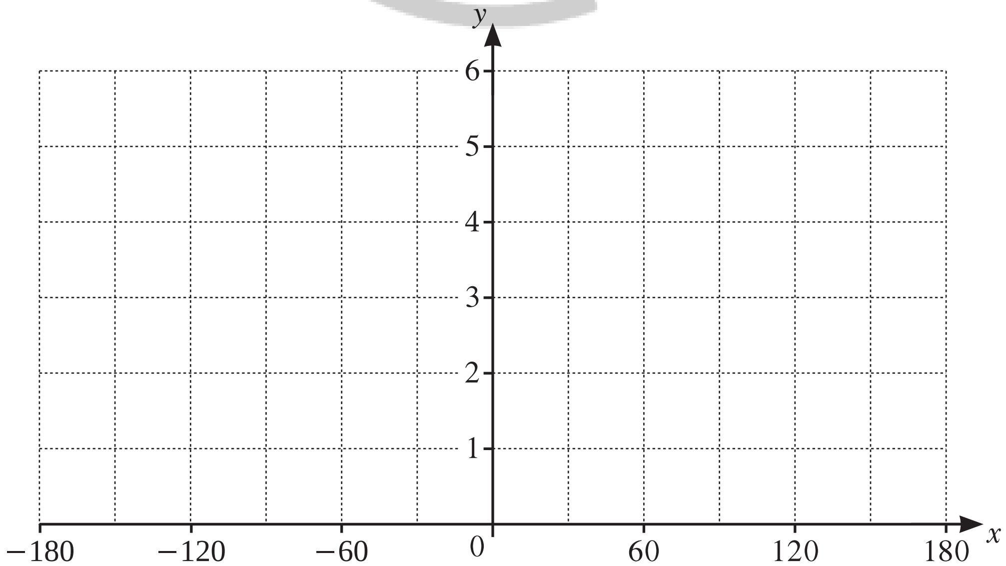


The diagram shows the graph of $y = |p(x)|$, where $p(x)$ is a cubic function. Find the two possible expressions for $p(x)$. [3]

2 **(a)** Write down the amplitude of $1 + 4\cos\left(\frac{x}{3}\right)$. [1]

(b) Write down the period of $1 + 4\cos\left(\frac{x}{3}\right)$. [1]

(c) On the axes below, sketch the graph of $y = 1 + 4\cos\left(\frac{x}{3}\right)$ for $-180° \leqslant x° \leqslant 180°$.



[3]

3 **(a)** Write $\dfrac{\sqrt{p}\left(qr^2\right)^{\frac{1}{3}}}{\left(q^3p\right)^{-1}r^3}$ in the form $p^aq^br^c$, where a, b and c are constants. [3]

(b) Solve $6x^{\frac{2}{3}}-5x^{\frac{1}{3}}+1=0$. [3]

4 It is given that $y = \dfrac{\tan 3x}{\sin x}$.

(a) Find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \dfrac{\pi}{3}$. [4]

(b) Hence find the approximate change in y as x increases from $\dfrac{\pi}{3}$ to $\dfrac{\pi}{3} + h$, where h is small. [1]

(c) Given that x is increasing at the rate of 3 units per second, find the corresponding rate of change in y when $x = \dfrac{\pi}{3}$, giving your answer in its simplest surd form. [2]

5 **(a)** **(i)** Find how many different 4-digit numbers can be formed using the digits 1, 3, 4, 6, 7 and 9. Each digit may be used once only in any 4-digit number. [1]

(ii) How many of these 4-digit numbers are even and greater than 6000? [3]

(b) A committee of 5 people is to be formed from 6 doctors, 4 dentists and 3 nurses. Find the number of different committees that could be formed if

(i) there are no restrictions, [1]

(ii) the committee contains at least one doctor, [2]

(iii) the committee contains all the nurses. [1]

6 A particle P is initially at the point with position vector $\begin{pmatrix}30\\10\end{pmatrix}$ and moves with a constant speed of $10\,\text{ms}^{-1}$ in the same direction as $\begin{pmatrix}-4\\3\end{pmatrix}$.

(a) Find the position vector of P after t s. [3]

As P starts moving, a particle Q starts to move such that its position vector after t s is given by $\begin{pmatrix}-80\\90\end{pmatrix}+t\begin{pmatrix}5\\12\end{pmatrix}$.

(b) Write down the speed of Q. [1]

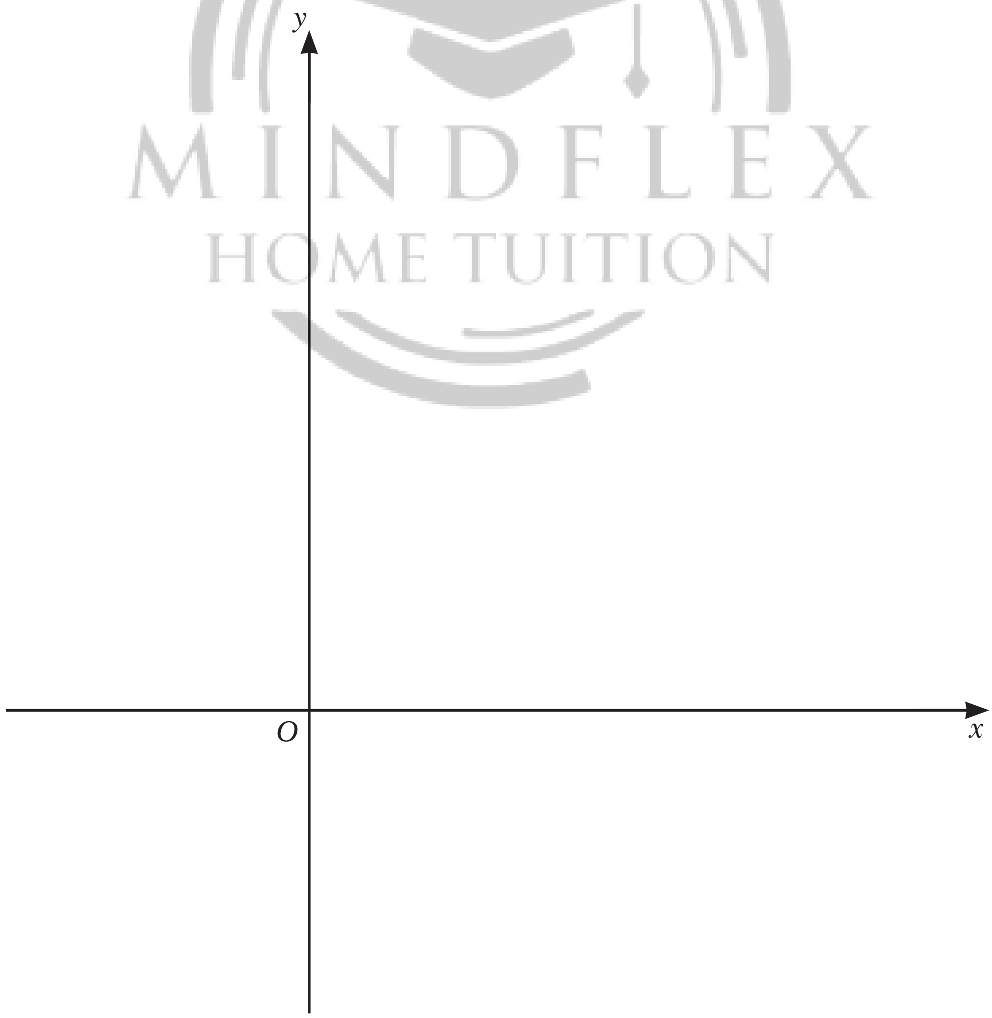
(c) Find the exact distance between P and Q when $t = 10$, giving your answer in its simplest surd form. [3]

7 It is given that $f(x) = 5\ln(2x+3)$ for $x > -\dfrac{3}{2}$.

(a) Write down the range of f. [1]

(b) Find f^{-1} and state its domain. [3]

(c) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each curve and state the intercepts on the coordinate axes.



[5]

[Turn over

8 **(a)** **(i)** Show that $\dfrac{1}{(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta)} = \sec^2\theta$. [4]

(ii) Hence solve $(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta) = \dfrac{3}{4}$ for $-180° \leqslant \theta \leqslant 180°$. [4]

(b) Solve $\sin\left(3\phi + \frac{2\pi}{3}\right) = \cos\left(3\phi + \frac{2\pi}{3}\right)$ for $0 \leqslant \phi \leqslant \frac{2\pi}{3}$ radians, giving your answers in terms of π. [4]

9 **(a)** Given that $\int_1^a \left(\frac{1}{x} - \frac{1}{2x+3}\right) \mathrm{d}x = \ln 3$, where $a > 0$, find the exact value of a, giving your answer in simplest surd form. [6]

(b) Find the exact value of $\int_0^{\frac{\pi}{3}}\left(\sin\left(2x+\frac{\pi}{3}\right)-1+\cos 2x\right)\mathrm{d}x$. [5]

10 **(a)** An arithmetic progression has a second term of 8 and a fourth term of 18. Find the least number of terms for which the sum of this progression is greater than 1560. [6]

(b) A geometric progression has a sum to infinity of 72. The sum of the first 3 terms of this progression is $\frac{333}{8}$.

(i) Find the value of the common ratio. [5]

(ii) Hence find the value of the first term. [1]

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ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

 [Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

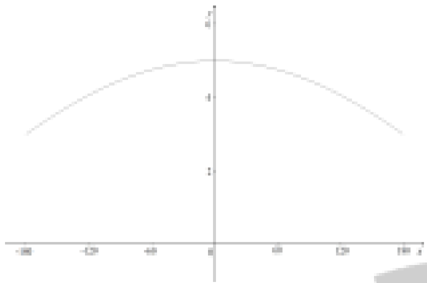
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1	$y = \pm 3(x+2)(x+1)(x-4)$	**3**	**B1** for 3 **B1** for $(x+2)(x+1)(x-4)$ **B1** for $\pm$
2(a)	4	**B1**	
2(b)	1080° or 6π	**B1**	
2(c)		**3**	**B1** for shape, it must be symmetrical about the *y*-axis. **B1**for *y*-intercept of 5 **B1** for $(\pm 180^{\circ}, 3)$
3(a)	$a = \frac{3}{2}$ or $p^{\frac{3}{2}}$	**B1**	
	$b = \frac{10}{3}$ or $q^{\frac{10}{3}}$	**B1**	
	$c = -\frac{7}{3}$ or $r^{-\frac{7}{3}}$	**B1**	
3(b)	$\left(3x^{\frac{1}{3}} - 1\right)\left(2x^{\frac{1}{3}} - 1\right) = 0$	**M1**	For recognising as a quadratic in $x^{\frac{1}{3}}$ and attempt to solve to obtain $x^{\frac{1}{3}} = k$
	$x^{\frac{1}{3}} = \frac{1}{3}$, $x^{\frac{1}{3}} = \frac{1}{2}$ leading to $x = \frac{1}{27}$ or 0.0370 $x = \frac{1}{8}$ or 0.125	**2**	**Dep M1** for a valid method of solving $x^{\frac{1}{3}} = k$ where $k > 0$ **A1** for both
4(a)	$\frac{dy}{dx} = \frac{\sin x \times 3\sec^2 3x - \tan 3x \cos x}{\sin^2 x}$	**3**	**B1** for $3\sec^2 3x$ **M1** for differentiation of a quotient or equivalent product **A1** for all other terms apart from $3\sec^2 3x$ correct
	When $x = \frac{\pi}{3}$ $\frac{dy}{dx} = 2\sqrt{3}$	**A1**	
4(b)	$2\sqrt{3}h$	**B1**	**FT** on *their* answer to **(a)**

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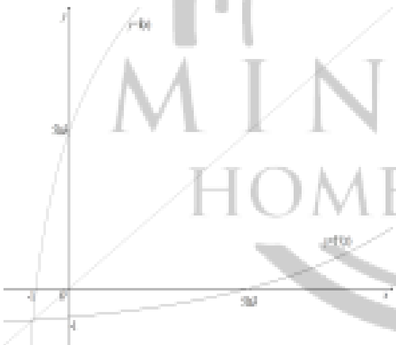
Question	Answer	Marks	Guidance
4(c)	$\frac{dy}{dx} \times \frac{dx}{dt} = \frac{dy}{dt}$ $2\sqrt{3} \times 3 = \frac{dy}{dt}$	M1	For correct use of rates of change using *their* answer to **(a)**
	$\frac{dy}{dt} = 6\sqrt{3}$	A1	
5(a)(i)	360	B1	
5(a)(ii)	Starts with 6: $1 \times 4 \times 3 \times 1 \ = 12$	B1	
	Starts with 7 or 9 : $= 2 \times 4 \times 3 \times 2$ $= 48$	B1	
	Total = 60	B1	
	Alternative		
	Ending in 4: $\frac{1}{6} \times 360 \times \frac{3}{5} = 36$	(B1)	Allow unsimplified
	Ending in 6: $\frac{1}{6} \times 360 \times \frac{2}{5} = 24$	(B1)	Allow unsimplified
	Total = 60	(B1)	
5(b)(i)	1287	B1	
5(b)(ii)	$1287 - {}^7C_5$ or 1 doctor: 210 2 doctors: 525 3 doctors: 420 4 doctors: 105 5 doctors: 1	M1	For *their* **(b)(i)** $-{}^7C_5$ or listing all the other separate cases which must be evaluated, allow 1 error
	1266	A1	
5(b)(iii)	45	B1	
6(a)	Velocity vector = $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$	2	**M1** for obtaining 5
	$\begin{pmatrix} 30 \\ 10 \end{pmatrix} + \begin{pmatrix} -8 \\ 6 \end{pmatrix} t$	B1	**FT** for $\begin{pmatrix} 30 \\ 10 \end{pmatrix} + (\textit{their} \text{ velocity vector})t$
6(b)	13	B1	

Question	Answer	Marks	Guidance
6(c)	$P:\begin{pmatrix}-50\\70\end{pmatrix}$ $Q:\begin{pmatrix}-30\\210\end{pmatrix}$	M1	Using $t=10$ to find position vector of each particle
	$\sqrt{20^2+140^2}$	M1	**Dep** on previous M mark, for use of Pythagoras on difference of the 2 position vectors
	$100\sqrt{2}$	A1	
7(a)	$\mathrm{f}\in\mathbb{R}$	B1	Allow *y* but not *x*
7(b)	$x=5\ln(2y+3)$ $\mathrm{e}^{\frac{x}{5}}=2y+3$	M1	For a complete attempt to obtain inverse
	$\mathrm{f}^{-1}(x)=\dfrac{\mathrm{e}^{\frac{x}{5}}-3}{2}$	A1	Must be using correct notation
	Domain $x\in\mathbb{R}$	B1	**FT** on *their* **(a)**. Must be using correct notation
7(c)		5	**B1** for shape of $y=\mathrm{f}(x)$ **B1** for shape of $y=\mathrm{f}^{-1}(x)$ **B1** for $5\ln 3$ or 5.5 and -1 on both axes for $y=\mathrm{f}(x)$ **B1** for $5\ln 3$ or 5.5 and -1 on both axes for $y=\mathrm{f}^{-1}(x)$ **B1** All correct, with apparent symmetry which may be implied be previous 2 B marks or by inclusion of $y=x$, and implied asymptotes, may have one or two points of intersection
8(a)(i)	$\dfrac{1}{\left(1+\dfrac{1}{\sin\theta}\right)\left(\sin\theta-\sin^2\theta\right)}$	B1	For use of $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$, may be implied
	$\dfrac{1}{\sin\theta+1-\sin\theta-\sin^2\theta}$	M1	For expansion of brackets
	$\dfrac{1}{\cos^2\theta}$	M1	For simplification and use of identity
	$\sec^2\theta$	A1	For final result, must see $\dfrac{1}{\cos^2\theta}$

Question	Answer	Marks	Guidance
8(a)(ii)	$\cos^2\theta = \frac{3}{4}$	**B1**	For relating to and making use of **(a)**
	$\cos\theta = \pm\frac{\sqrt{3}}{2}$	**M1**	For attempt to solve, may be implied by one correct solution
	$\theta = -150^\circ,\ -30^\circ,\ 30^\circ,\ 150^\circ$	**2**	**A1** for any correct pair **A1** for a second correct pair and no extra solutions within the range
8(b)	$\tan\left(3\phi + \frac{2\pi}{3}\right) = 1$	**B1**	
	$3\phi + \frac{2\pi}{3} = \frac{\pi}{4},\ \frac{5\pi}{4},\ \frac{9\pi}{4}$ $3\phi = \frac{7\pi}{12},\ \frac{19\pi}{12}$	**M1**	For correct order of operations
	$\phi = \frac{7\pi}{36}$	**A1**	
	$\phi = \frac{19\pi}{36}$	**A1**	
9(a)	$\left[\ln x - \frac{1}{2}\ln(2x+3)\right]_1^a$	**2**	**B1** for $\ln x$ **B1** for $\frac{1}{2}\ln(2x+3)$
	$\ln a - \frac{1}{2}\ln(2a+3) + \frac{1}{2}\ln 5$	**M1**	For correct application of limits, must have at least one **B1**
	$\ln a\sqrt{\frac{5}{2a+3}}$	**M1**	**Dep** on previous M mark, for application of log laws
	$5a^2 - 18a - 27 = 0$	**M1**	**Dep** on previous M mark for equating to $\ln 3$ and simplification to a 3 term quadratic = 0
	$a = \frac{9+6\sqrt{6}}{5}$	**A1**	Must have one solution only

Question	Answer	Marks	Guidance
9(b)	$-\frac{1}{2}\cos\left(2x+\frac{\pi}{3}\right)+\frac{1}{2}\sin 2x - x$	3	**B1** for $-\frac{1}{2}\cos\left(2x+\frac{\pi}{3}\right)$ **B1** for $+\frac{1}{2}\sin 2x$ **B1** for $-x$
	$\left(-\frac{1}{2}\cos\pi+\frac{1}{2}\sin\frac{2\pi}{3}-\frac{\pi}{3}\right)-\left(-\frac{1}{2}\cos\frac{\pi}{3}\right)$	M1	For correct use of limits in *their* integral, must have at least one **B1** term
	$\frac{3}{4}+\frac{\sqrt{3}}{4}-\frac{\pi}{3}$	A1	
10(a)	$a+d=8$ $a+3d=18$	2	**B1** for both equations **M1** for attempt to solve *their* equations
	$a=3,\ \ d=5$	A1	For both
	$\frac{n}{2}(6+(n-1)5)>1560$	M1	For correct use of sum formula with *their a* and *d*, allow equality
	$5n^2+n-3120>0$	M1	For attempt to solve, allow equality, to obtain at least one critical value
	Positive critical value 24.9 25terms	A1	
10(b)(i)	$\frac{a}{1-r}=72$ and either $a+ar+ar^2=\frac{333}{8}$ or $\frac{a(1-r^3)}{1-r}=\frac{333}{8}$	B1	For both
	$a=72(1-r)$ and $a(1+r+r^2)=\frac{333}{8}$ oe $72(1-r)(1+r+r^2)=\frac{333}{8}$ or $72(1-r^3)=\frac{333}{8}$	M1	For attempt to obtain an equation in terms of *r* only
	$1-r^3=\frac{333}{576}$	A1	
	$r=0.75$	2	**M1** for attempt to solve *their* equation in *r*

Question	Answer	Marks	Guidance
10(b)(ii)	$a = 18$	**B1**	**FT** on their r provided $\lvert r \rvert < 1$

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CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

8320369122

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **October/November 2020**

2 hours

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- Do **not** write on any bar codes.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
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Mathematical Formulae

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Quadratic Equation

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Solve the inequality $|3x+2| > 8+x$. [3]

2 Find the coordinates of the points of intersection of the curve $x^2+xy=9$ and the line $y=\frac{2}{3}x-2$. [5]

3 Write $3\lg x + 2 - \lg y$ as a single logarithm. [3]

4 It is given that $y = \ln(\sin x + 3\cos x)$ for $0 < x < \frac{\pi}{2}$.

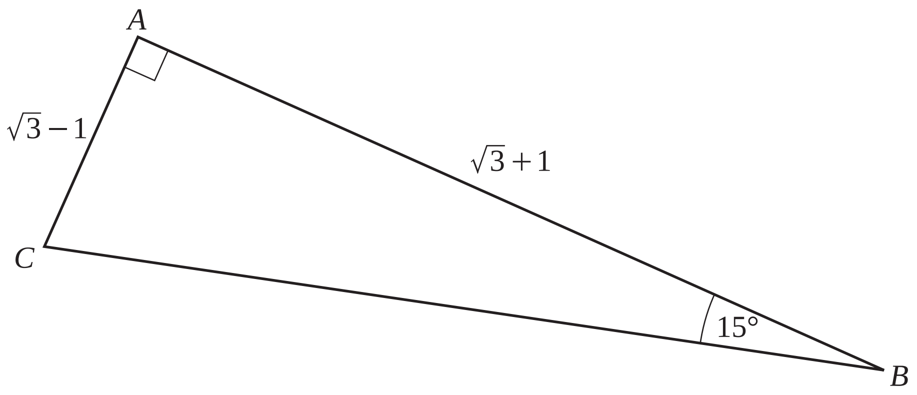
(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Find the value of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$. [3]

5 The first three terms in the expansion of $(a+bx)^5(1+x)$ are $32-208x+cx^2$. Find the value of each of the integers a, b and c. [7]

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question all lengths are in centimetres.



In the diagram above $AC = \sqrt{3}-1$, $AB = \sqrt{3}+1$, angle $ABC = 15°$ and angle $CAB = 90°$.

(a) Show that $\tan 15° = 2-\sqrt{3}$. [3]

(b) Find the exact length of BC. [2]

7 DO NOT USE A CALCULATOR IN THIS QUESTION.

$$p(x) = 2x^3 - 3x^2 - 23x + 12$$

(a) Find the value of $p\left(\frac{1}{2}\right)$. [1]

(b) Write $p(x)$ as the product of three linear factors and hence solve $p(x) = 0$. [5]

8 The population P, in millions, of a country is given by $P = A \times b^t$, where t is the number of years after January 2000 and A and b are constants. In January 2010 the population was 40 million and had increased to 45 million by January 2013.

(a) Show that $b = 1.04$ to 2 decimal places and find A to the nearest integer. [4]

(b) Find the population in January 2020, giving your answer to the nearest million. [1]

(c) In January of which year will the population be over 100 million for the first time? [3]

9 A particle moves in a straight line such that, t seconds after passing a fixed point O, its displacement from O is s m, where $s = e^{2t} - 10e^{t} - 12t + 9$.

(a) Find expressions for the velocity and acceleration at time t. [3]

(b) Find the time when the particle is instantaneously at rest. [3]

(c) Find the acceleration at this time. [2]

10 The gradient of the normal to a curve at the point (x, y) is given by $\frac{x}{x+1}$.

(a) Given that the curve passes through the point (1, 4), show that its equation is $y = 5 - \ln x - x$. [5]

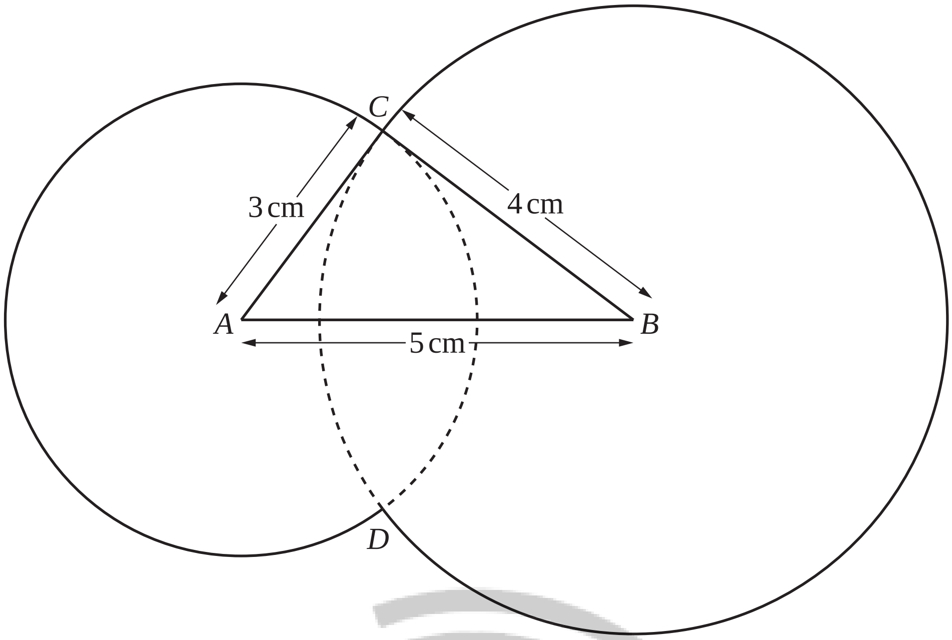
(b) Find, in the form $y = mx + c$, the equation of the tangent to the curve at the point where $x = 3$. [3]

11 The equation of a curve is $y = x\sqrt{16 - x^2}$ for $0 \leqslant x \leqslant 4$.

(a) Find the exact coordinates of the stationary point of the curve. [6]

(b) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(16-x^2\right)^{\frac{3}{2}}$ and hence evaluate the area enclosed by the curve $y = x\sqrt{16-x^2}$ and the lines $y = 0$, $x = 1$ and $x = 3$. [5]

12



The diagram shows a shape consisting of two circles of radius 3 cm and 4 cm with centres *A* and *B* which are 5 cm apart. The circles intersect at *C* and *D* as shown. The lines *AC* and *BC* are tangents to the circles, centres *B* and *A* respectively. Find

(a) the angle *CAB* in radians, [2]

(b) the perimeter of the whole shape, [4]

(c) the area of the whole shape. [4]

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Cambridge Assessment International Education

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ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **October/November 2020**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **7** printed pages.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Partial Marks
1	$3x+2>8+x \rightarrow x>3$	**B1**	
	$-3x-2>8+x$	**M1**	Correct inequality oe
	$x<-2.5$	**A1**	
2	$x^2+x\left(\frac{2}{3}x-2\right)=9$	**M1**	Eliminate y
	$5x^2-6x-27=0$	**A1**	
	$(x-3)(5x+9)=0$	**M1**	Factorise or formula
	(3, 0)	**A1**	Or both x values
	$\left(-\frac{9}{5},-\frac{16}{5}\right)$	**A1**	
3	Uses $\lg 100=2$ or $3\lg x=\lg x^3$.	**B1**	
	Uses $\lg a+\lg b=\lg ab$ or $\lg a-\lg b=\lg\left(\frac{a}{b}\right)$	**B1**	
	$\lg\left(\frac{100x^3}{y}\right)$	**B1**	Correct final answer
4(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\cos x-3\sin x}{\sin x+3\cos x}$	**3**	**M1** for attempt at chain rule must have function in numerator and denominator **A1** for denominator **A1** for numerator
(b)	$-2\cos x-3\cos x=\sin x-6\sin x$	**M1**	Expand and collect terms in $\sin x$ and $\cos x$
	$1=\tan x$	**M1**	Use $\frac{\sin x}{\cos x}=\tan x$
	$x=\frac{\pi}{4}$	**A1**	Must be radians
5	$a^5+5a^4bx+10a^3b^2x^2$	**2**	**B1** for powers or for coefficients
	$a^5+\left(a^5+5a^4b\right)x+\left(10a^3b^2+5a^4b\right)x^2$	**2**	**M1** for multiplying to obtain 5 terms **A1** for all correct
	$a^5=32 \rightarrow a=2$	**A1**	
	$32+80b=-208 \rightarrow b=-3$	**A1**	
	$10\times8\times9+5\times16\times-3=c \rightarrow c=480$	**A1**	

Question	Answer	Marks	Partial Marks
6(a)	$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	M1	Correct use of tan
	$\tan 15° = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$	M1	Multiply by $(\sqrt{3}-1)$
	$\tan 15° = 2-\sqrt{3}$	A1	**AG** So all working must be seen
6(b)	$(BC)^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2$	M1	Correct use of Pythagoras
	$BC = \sqrt{8}$ or $2\sqrt{2}$	A1	
7(a)	$\text{p}\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) - 3\left(\frac{1}{4}\right) - 23\left(\frac{1}{2}\right) + 12 = 0$	B1	Working must be seen
7(b)	$\text{p}(x) = (2x-1)(x^2 - x - 12)$	2	**M1** for terms x^2 and -12 **A1** for $-x$
	$\text{p}(x) = (2x-1)(x-4)(x+3)$	2	**M1** for solving quadratic **A1** for all three correct factors
	$\text{f}(x) = 0 \rightarrow x = \frac{1}{2}, 4, -3$	A1	
8(a)	$40 = A \times b^{10}$ and $45 = A \times b^{13}$	B1	
	$b^3 = \frac{45}{40}$	M1	Divide to find b^3.
	$b = 1.04$	A1	
	$A = 27$	A1	
8(b)	59	B1	$P = 27 \times 1.04^{20}$
8(c)	$100 = 27 \times 1.04^t$	M1	Insert P = 100 in their expression
	$t = \frac{\log\left(\frac{100}{27}\right)}{\log 1.04}$ oe	M1	Rearrange to make t the subject
	$t = 33.4 \rightarrow$ Year 2034	A1	
9(a)	$v = 2e^{2t} - 10e^t - 12$ $a = 4e^{2t} - 10e^t$	3	**M1** for correctly differentiating e^{2t}. **A1** for v correct **A1** for a correct

Question	Answer	Marks	Partial Marks
9(b)	$v = 0 \to e^{2t} - 5e^t - 6 = 0$ $\to (e^t + 1)(e^t - 6) = 0$	**M1**	Factorise quadratic Solve and discard $e^t = -1$
	$e^t = 6$	**A1**	
	$t = \ln 6 = 1.79$	**A1**	
9(c)	$t = \ln 6 \to a = 4 \times 36 - 10 \times 6 = 84$	**2**	**M1** for inserting *their* value of t into a
10(a)	$\frac{dy}{dx} = -\frac{(1+x)}{x} = -\left(\frac{1}{x} + 1\right)$	**2**	**M1** for using $m_1 \times m_2 = -1$
	$y = -\ln x - x + C$	**2**	**M1** for integrating $\frac{1}{x}$ **A1** for all correct including C
	$4 = -\ln 1 - 1 + C$ $C = 5 \to y = 5 - \ln x - x$	**A1**	Insert (1, 4) and arrive at correct answer. AG
10(b)	$x = 3 \to y = 2 - \ln 3$ and $\frac{dy}{dx} = -\frac{1}{3} - 1 = -\frac{4}{3}$	**B1**	
	$\frac{y - (2 - \ln 3)}{x - 3} = -\frac{4}{3}$	**M1**	
	$y = -\frac{4}{3}x + 6 - \ln 3$ or $y = -1.33x + 4.90$	**A1**	
11(a)	$\frac{dy}{dx} = x \times \frac{1}{2}(16 - x^2)^{-\frac{1}{2}} \times (-2x) + (16 - x^2)^{\frac{1}{2}}$	**3**	**B1** for $\frac{d}{dx}(16 - x^2)^{\frac{1}{2}}$ $= \frac{1}{2}(16 - x^2)^{-\frac{1}{2}} \times (-2x)$ **M1** for product rule **A1** for all correct
	$\frac{dy}{dx} = 0 \to (16 - x^2)^{\frac{1}{2}} = \frac{x^2}{(16 - x^2)^{\frac{1}{2}}}$ $x^2 = 8$ $(2\sqrt{2}, 8)$	**3**	**M1** for setting $\frac{dy}{dx} = 0$ and attempt to solve **M1** for obtaining $x^2 = k$ **A1**

Question	Answer	Marks	Partial Marks
11(b)	$\frac{3}{2}\left(16-x^2\right)^{\frac{1}{2}}\times(-2x)$	2	**M1** for attempt at chain rule **A1** for all correct unsimplified
	Area = $\int_1^3 x\left(16-x^2\right)^{\frac{1}{2}}\,dx = \left[-\frac{1}{3}\left(16-x^2\right)^{\frac{3}{2}}\right]_1^3$ $=-\frac{1}{3}\left[7^{\frac{3}{2}}-15^{\frac{3}{2}}\right]=13.2$	3	**M1** for obtaining $k\left(16-x^2\right)^{\frac{3}{2}}$ **A1** for obtaining $k=-\frac{1}{3}$ **A1** for 13.2
12(a)	$\tan CAB=\frac{4}{3}$	M1	Correct use of tan oe
	$CAB=0.927$	A1	isw
12(b)	Angle *CBD* = $2\left(\frac{\pi}{2}-0.927\right)=1.287$	B1	
	Perimeter $=3(2\pi-2\times0.927)+4(2\pi-1.287)$ $=13.287+19.985$ $=33.3$	3	**M1** for correct plan of two arcs **A1** for either arc **A1**
12(c)	Area of two right-angled triangles = $\frac{1}{2}\times3\times4\times2=12$	B1	
	Area of Sectors = $\frac{3^2}{2}(2\pi-2\times0.927)+\frac{4^2}{2}(2\pi-1.287)$ $=19.93+39.97$ Total = 71.9	3	**M1** for correct plan of two sectors plus triangles **A1** for either sector **A1**